Tully Fisher Relation

Connection between absolute magnitude (luminosity) and circular velocity or velocity width

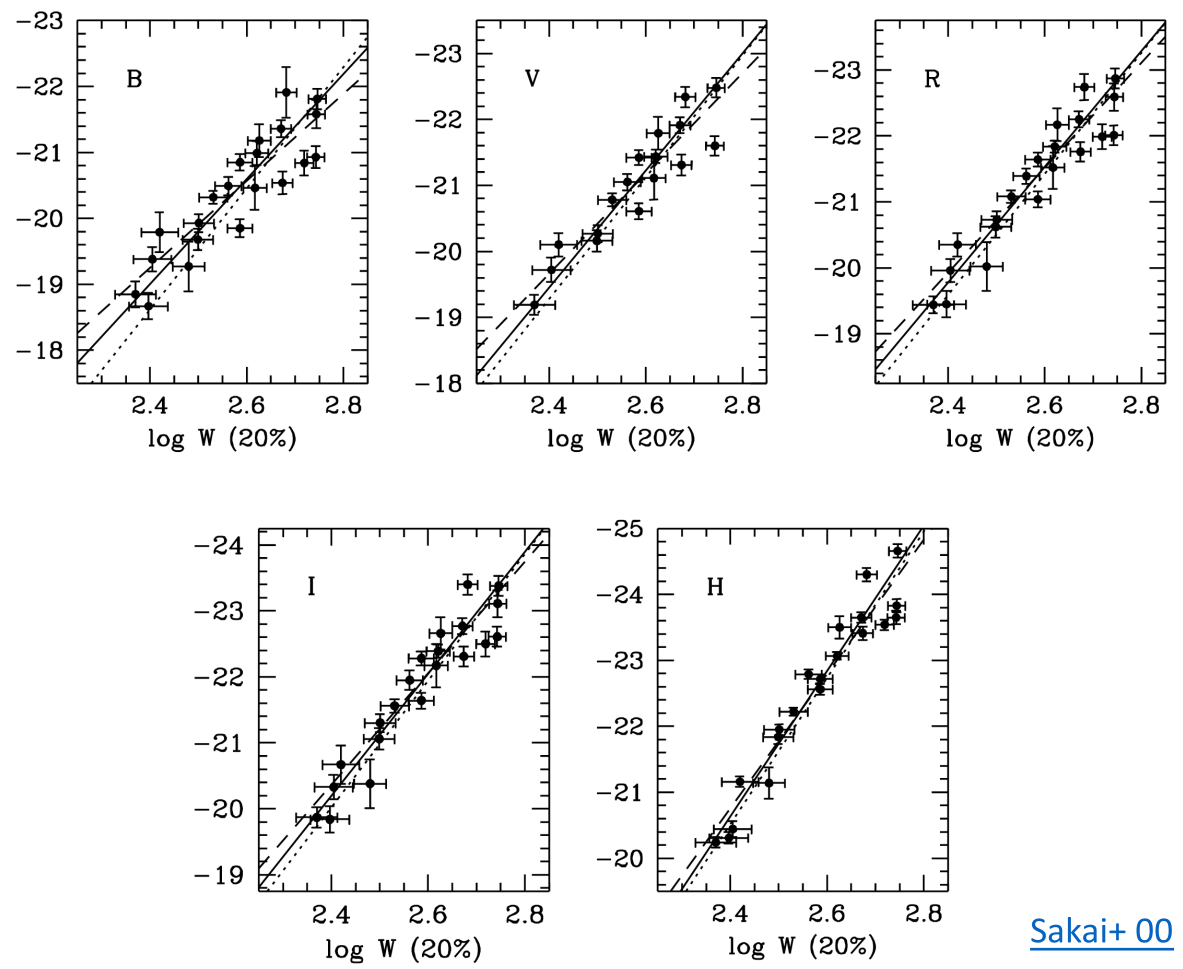
$$M = -A[\log(W_{20}) - 2.5] + ZP$$

A : slope
ZP : abs mag of a galaxy with $\log(W_{20}) = 2.5$

Slope and scatter both provide important astrophysical constraints. As we measure the TF relation in redder bands:

- Slope gets steeper
- Scatter (σ) goes down

Calibrated TF relations from HST Cepheid distances



Sakai+ 00

$$B_T^c = -(7.97 \pm 0.72)(\log W_{20}^c - 2.5) - (19.80 \pm 0.11)$$
$$V_T^c = -(8.87 \pm 0.83)(\log W_{20}^c - 2.5) - (20.34 \pm 0.12)$$
$$R_T^c = -(8.78 \pm 0.72)(\log W_{20}^c - 2.5) - (20.65 \pm 0.11)$$
$$I_T^c = -(9.24 \pm 0.75)(\log W_{20}^c - 2.5) - (21.12 \pm 0.12)$$
$$H_{-0.5}^c = -(11.03 \pm 0.86)(\log W_{20}^c - 2.5) - (21.74 \pm 0.14)$$

$\sigma \approx 0.3 - 0.5$

$\sigma \approx 0.1$

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Systematic uncertainties in Tully-Fisher:

- **Dust corrections**: are we seeing all the light? Dust may be obscuring it. *Solution: use the near-IR, where dust obscuration is minimal.*

- **Stellar populations**: massive young stars can produce a lot of light for short time; recent star formation can skew the measurement of the overall light from all the stars. *Solution: use the near-IR, which is less sensitive to massive blue stars.*

In near-IR, scatter in well-defined samples is consistent with purely observational error: *no intrinsic scatter!?!*

But also systematic uncertainties due to

- Which **circular velocity measurement** to use? (W_{50}, W_{20}, V_{max}, V_{flat}, etc)

- Uncertain **inclination corrections** to go from measured velocity width to true circular velocity.

Tully Fisher Relation: Implications from Scaling Relations

Total mass ($\mathcal{M}_{tot}$) couples with velocity (V_c) and size (R): $V_c^2 = G\mathcal{M}_{tot}/R \longrightarrow \mathcal{M}_{tot} \propto V_c^2 R$

Luminosity (L_{tot}) couples with luminosity density (I_0) and size: $L_{tot} \propto I_0 R^2$

Total Mass couples with Luminosity via a ***total*** mass-to-light ratio: $\mathcal{M}_{tot} = \left(\frac{\mathcal{M}}{L}\right)_{tot} L_{tot} \longrightarrow \mathcal{M}_{tot} \propto \left(\frac{\mathcal{M}}{L}\right)_{tot} I_0 R^2$

Equate the two total mass tracers:

$$V_c^2 R \propto \left(\frac{\mathcal{M}}{L}\right)_{tot} I_0 R^2 \quad \longrightarrow \quad L_{tot} \propto \frac{V_c^4}{I_0 \left(\frac{\mathcal{M}}{L}\right)_{tot}^2}$$

Then convert to absolute mags:

$$M = -2.5 \log L_{tot} + C \quad \longrightarrow \quad M \propto -10 \log V_c$$

which would give a Tully-Fisher slope of -10, which is close to what we observe using infrared light.

But that ***only*** holds if $I_0 \left(\frac{M}{L}\right)_{tot}^2$ = constant across all spiral galaxies, meaning that stars (providing the light) and dark matter (providing total mass) are tightly coupled. *This is kinda crazy!*

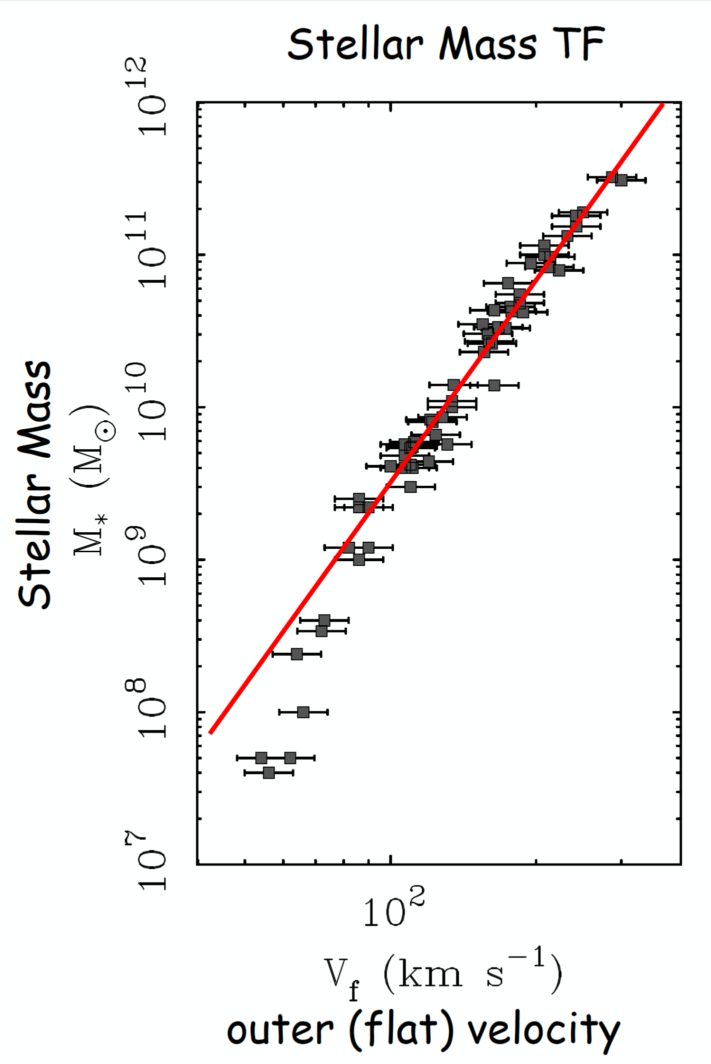
The Baryonic Tully-Fisher Relation (McGaugh 05, etc)

Instead of correlations between light and velocity, look at the connection between baryonic mass and velocity.

Convert light to stellar mass using stellar population models 👉

$$M_* = \left(\frac{M}{L}\right)_* L$$

Low luminosity galaxies deviate from TF, they are too low.



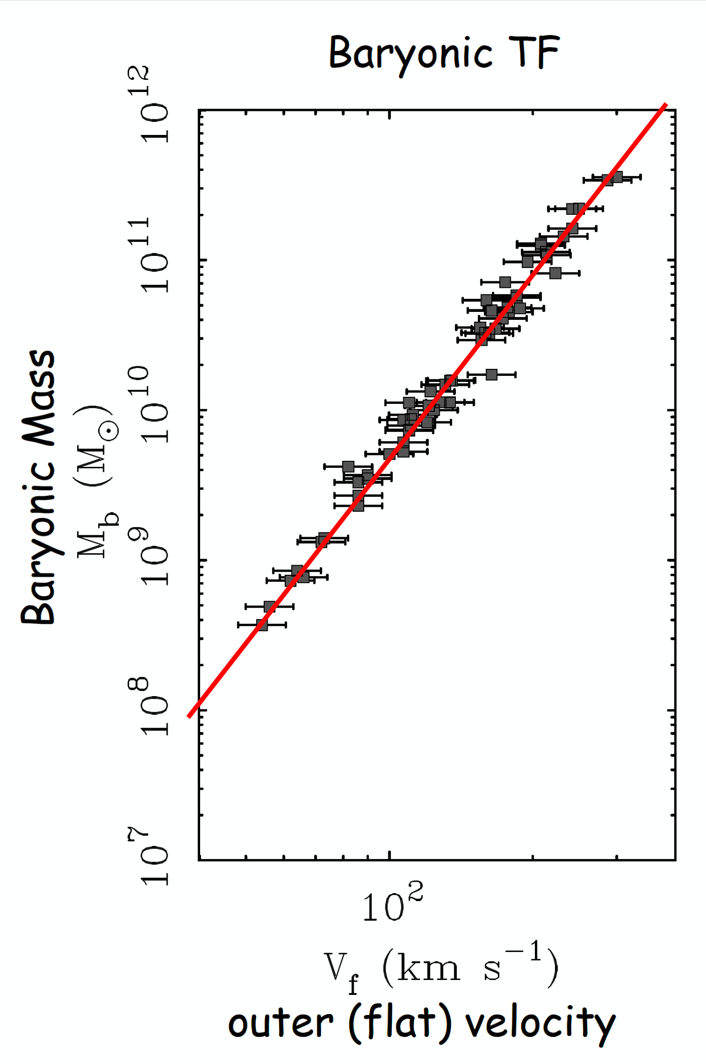
Add in gas mass 👉

$$M_b = M_* + M_{gas}$$

Wow.

slope= 3.94
± 0.07 (ran)
± 0.08 (sys)

scatter consistent with pure observational uncertainties.



Why Tully-Fisher is so important:

It demonstrates a very tight connection between baryonic matter (normal stuff) and gravitational motion.

- **Dark matter models:** very tight coupling between baryonic and non-baryonic matter. Doesn't come naturally from models. Solutions involve a lot of model-tuning.
- **Alternative gravity models:** more than just Newton/Einstein gravity.

It can be a useful tool for getting distances.

Circular velocity is distant-independent. Measure it for a galaxy, use TF to get the galaxy's absolute magnitude, couple that with the measured apparent magnitude, get a distance.

$$M = a \log V_c + b$$

It can be a useful tool for studying galaxy evolution.

When galaxies deviate from the mean, it tells you their kinematics are screwy or their mass-to-light ratio is different. If you see this systematically as a function of redshift, environment, or galaxy type, you learn about how galaxies differ in these respects.

Interpreting Rotation Curves: Back to dynamics

Fundamentally, we are building observable tracers of the underlying mass density of galaxies. To understand this, we need to tie it all together with a dynamical understanding of the relationships between mass, potential, and kinematics.

A galaxy has a mass distribution given by $\rho(\boldsymbol{x})$.

The gravitational potential is connected to density via Poisson's equation:

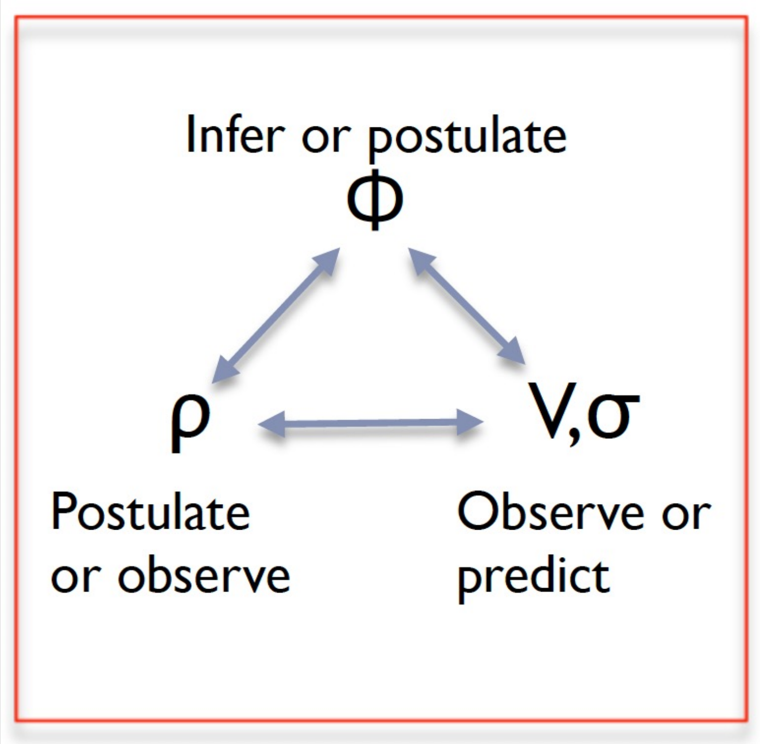
$$\nabla^2 \phi = 4\pi G \rho$$

Acceleration (i..e., motion) is derived from potential via

$$\mathbf{F} = \mathrm{m}\mathbf{a} = m\boldsymbol{\nabla\phi}$$

For **spherical** mass distributions, we can solve Poisson's equation as

$$\Phi(r) = -\underbrace{\frac{G\mathcal{M}(r)}{r}}_{\text{all mass inside } r \text{ acts as a point mass}} - \underbrace{G\int_r^\infty \frac{4\pi\rho(r)r^2}{r}dr}_{\text{outside } r\text{, integrate over mass shells to get potential}}$$



courtesy Matt Bershady

Simple Example: the constant density sphere

Density

$$\rho(r) = \rho_0 \ \text{ for } r < R_{max}$$

Derive mass interior to r:

$$\mathcal{M}(r) = \int_0^r \rho(r) 4\pi r^2 dr = 4\pi\rho_0 \int_0^r r^2 dr = \frac{4\pi r^3}{3}\rho_0$$

Derive circular velocity:

$$V_c^2(r) = \frac{G\mathcal{M}(r)}{r} \quad \text{or} \quad V_c(r) = \sqrt{\frac{4\pi G\rho_0}{3}} r$$

Derive potential:

$$\begin{aligned}\Phi(r) &= -\frac{G\mathcal{M}(r)}{r} - G\int_r^{R_{max}} \frac{4\pi\rho(r)r^2}{r} dr \\ &= -\frac{4\pi G\rho_0}{3} r^2 - 4\pi G\rho_0 \int_r^{R_{max}} r dr \\ &= -4\pi G\rho_0 \left[\frac{R_{max}^2}{2} - \frac{r^2}{6}\right]\end{aligned}$$

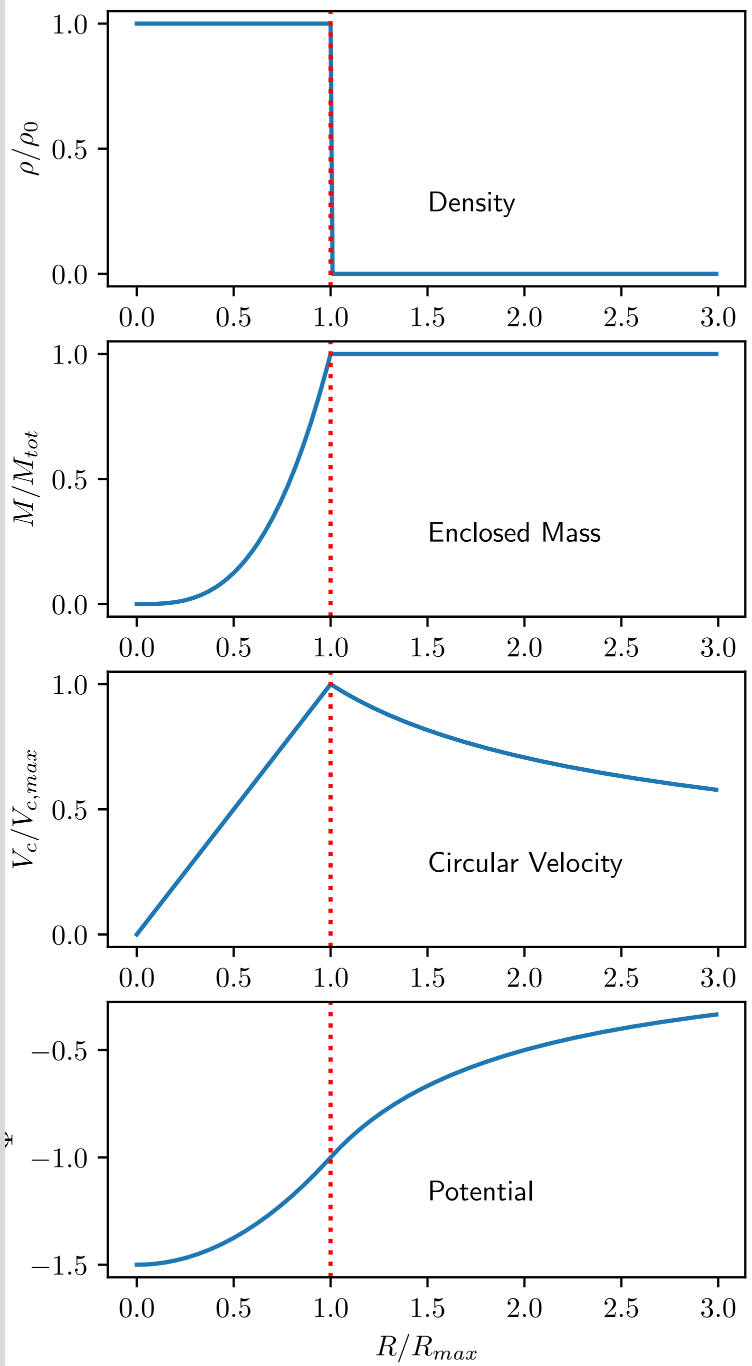
Outside R_{max} :

$$\rho(r) = 0$$

$$\mathcal{M}(r) = \frac{4\pi R_{max}^3}{3}\rho_0 = \mathcal{M}_{tot}$$

$$V_c(r) = \sqrt{\frac{G\mathcal{M}_{tot}}{r}}$$

$$\Phi(r) = -\frac{G\mathcal{M}_{tot}}{r}$$



Mass Modeling Rotation Curves

- Need to measure velocity (V_c) and know distance (to turn angular radial scale into physical scale).
- Need to have good surface brightness profile of the disk and bulge: $\mu_d(R)$ and $\mu_b(R)$
- Need to convert light to mass via a ***stellar*** mass-to-light ratio $(\mathcal{M}/L)_*$This depends on the stellar populations and will be different for the disk and bulge, and almost certainly also a function of radius.
- Need to measure gas content: neutral hydrogen is easy, need to correct for associated helium and molecular gas.
- Need to adopt a mass model for the dark matter halo, using theoretical profiles:

Pseudo-Isothermal Halo: $\rho(r) = \frac{\rho_0}{1+\left(\frac{r}{r_c}\right)^2}$

Navarro-Frenk-White: $\rho(r) = \frac{\rho_{crit}\delta_c}{\left(\frac{r}{r_s}\right)\left(1+\frac{r}{r_s}\right)^2}$

Einasto: $\rho(r) = \rho_c exp\left[-2n\left(\left(\frac{r}{r_c}\right)^{1/n} - 1\right)\right]$

Observe, correct for inclination, adopt distance

Observe μ_b, adopt $(M/L)_{*,b}$

$$V_c^2 = V_b^2 + V_d^2 + V_{gas}^2 + V_h^2$$

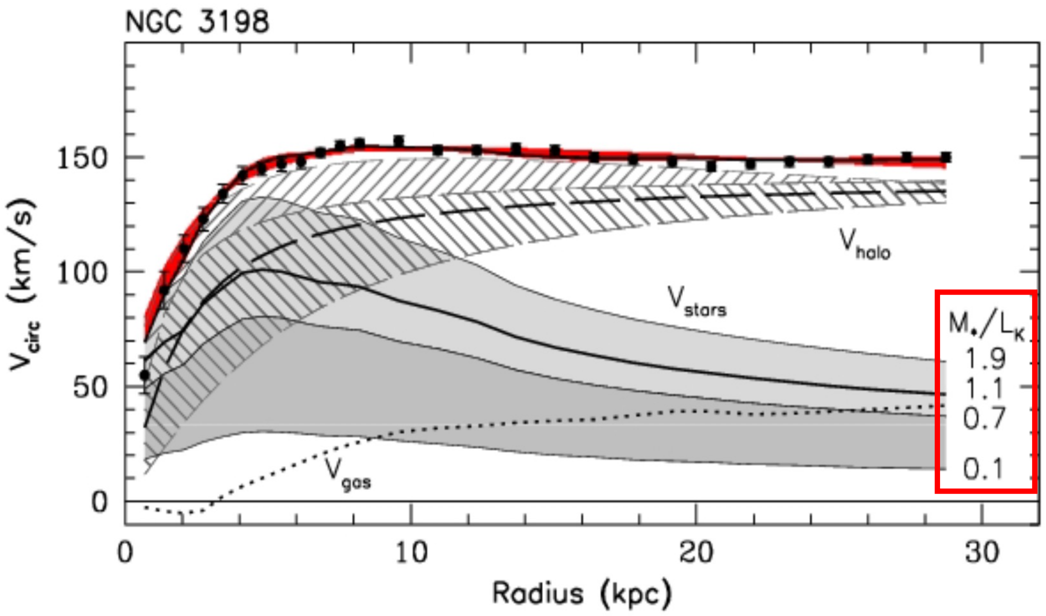
Observe μ_d, adopt $(M/L)_{*,d}$

Observe HI(+CO) make corrections

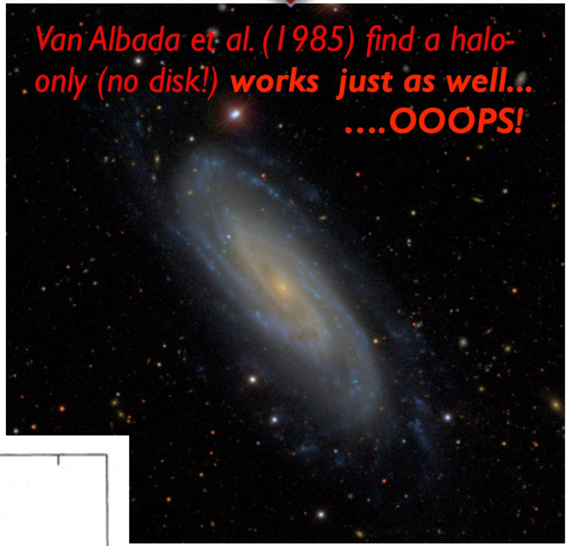
Adopt profile and fit parameters

The Disk-Halo Degeneracy: Best case

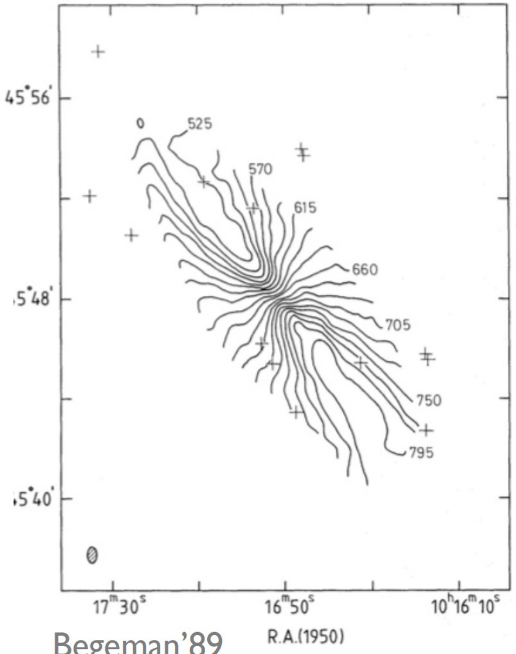
- Rotation curve decomposition constraints:



← Degenerate solutions…



SDSS gri



Begeman'89

…it doesn't get better than this

←HI velocity field

courtesy
Matt Bershady
(UWisc)

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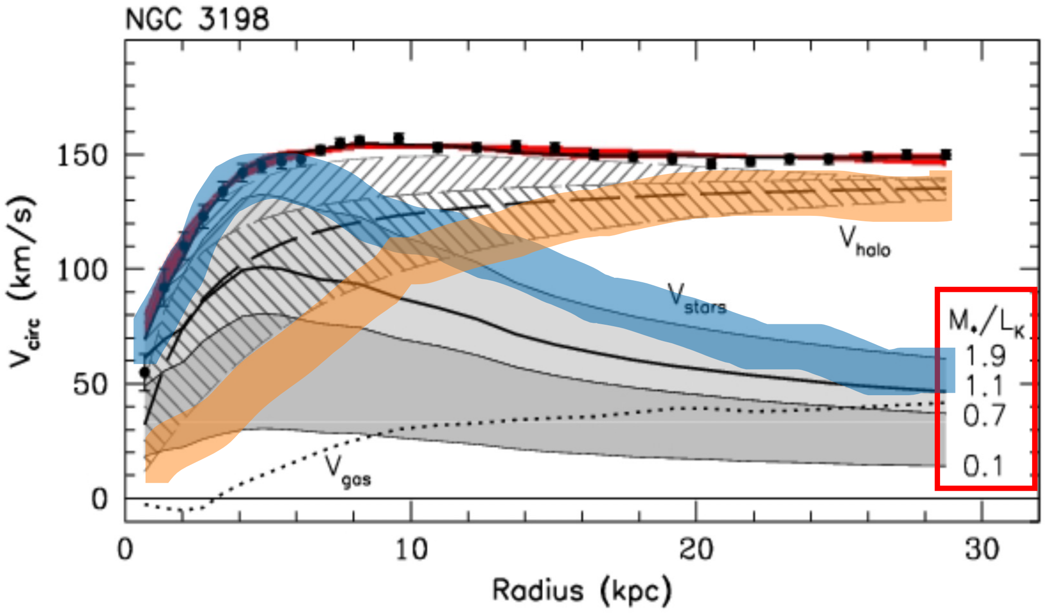
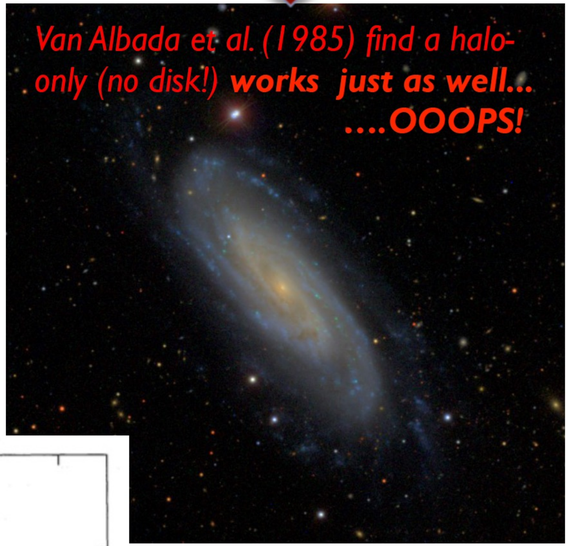


If $(M/L)_*$ is high, inner rotation is all due to disk mass. Outer parts still need halo component.

← ***Degenerate solutions…***



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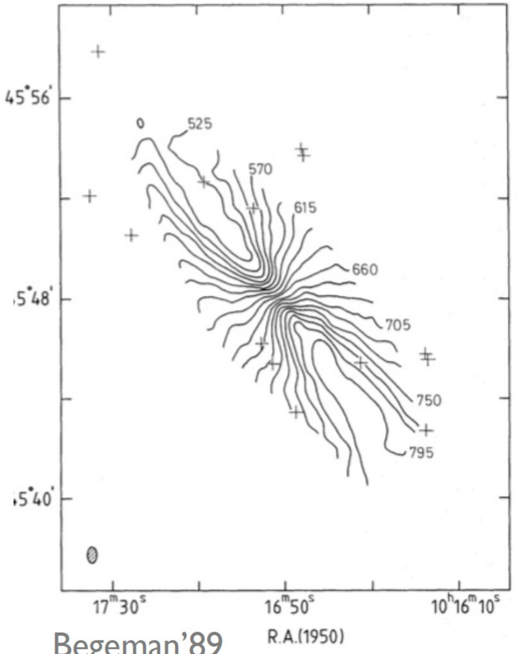

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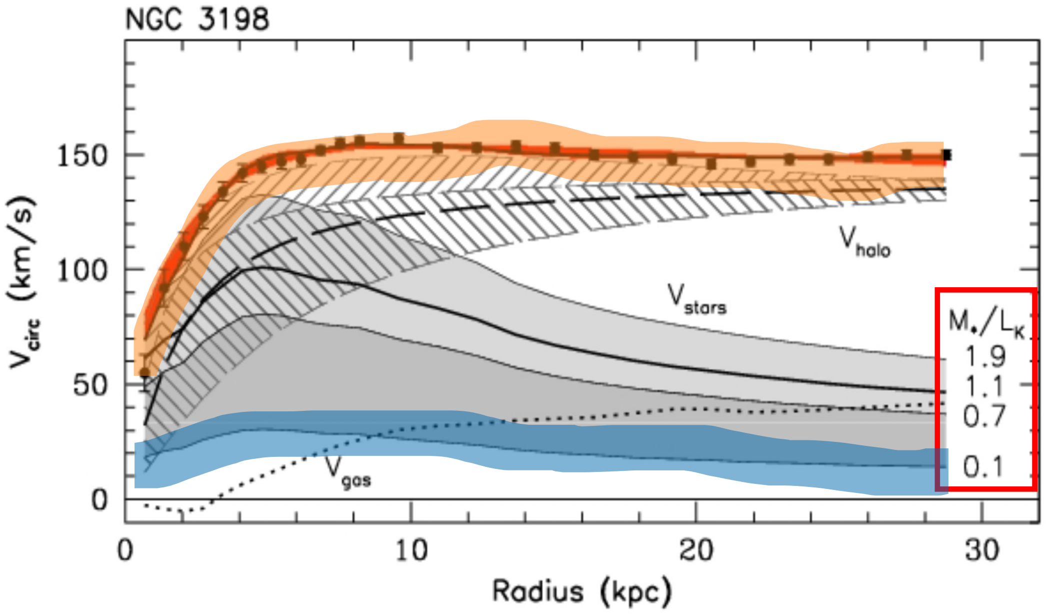

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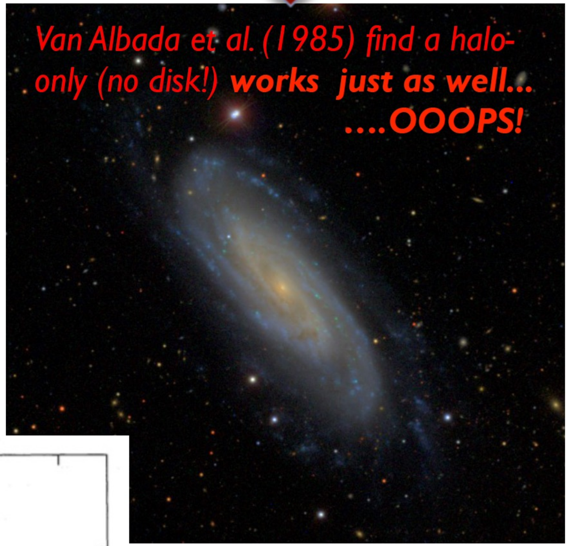

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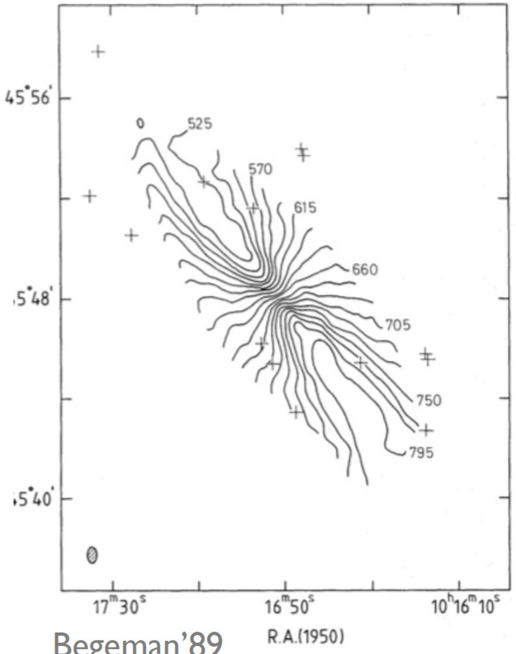

Begeman'89

…it doesn't get better than this

←HI velocity field

But if $(M/L)_*$ is low, the disk has less mass and the halo dominates throughout.

Disk/Halo Degeneracy: *we need to know $(M/L)_*$ very accurately to break this degeneracy!*

courtesy
Matt Bershady
(UWisc)

Spiral structure in disks

Types of spirals:

Grand design: 2 well-defined, symmetric spiral arms.

Flocculent: spiral arm “fragments”, not continuous

Multiple arms: 3, 4, etc

Barred spirals

M81 (Adam Block)

Spiral structure in disks

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Barred spirals

NGC 5054 (Michael Sidonio)

Spiral structure in disks

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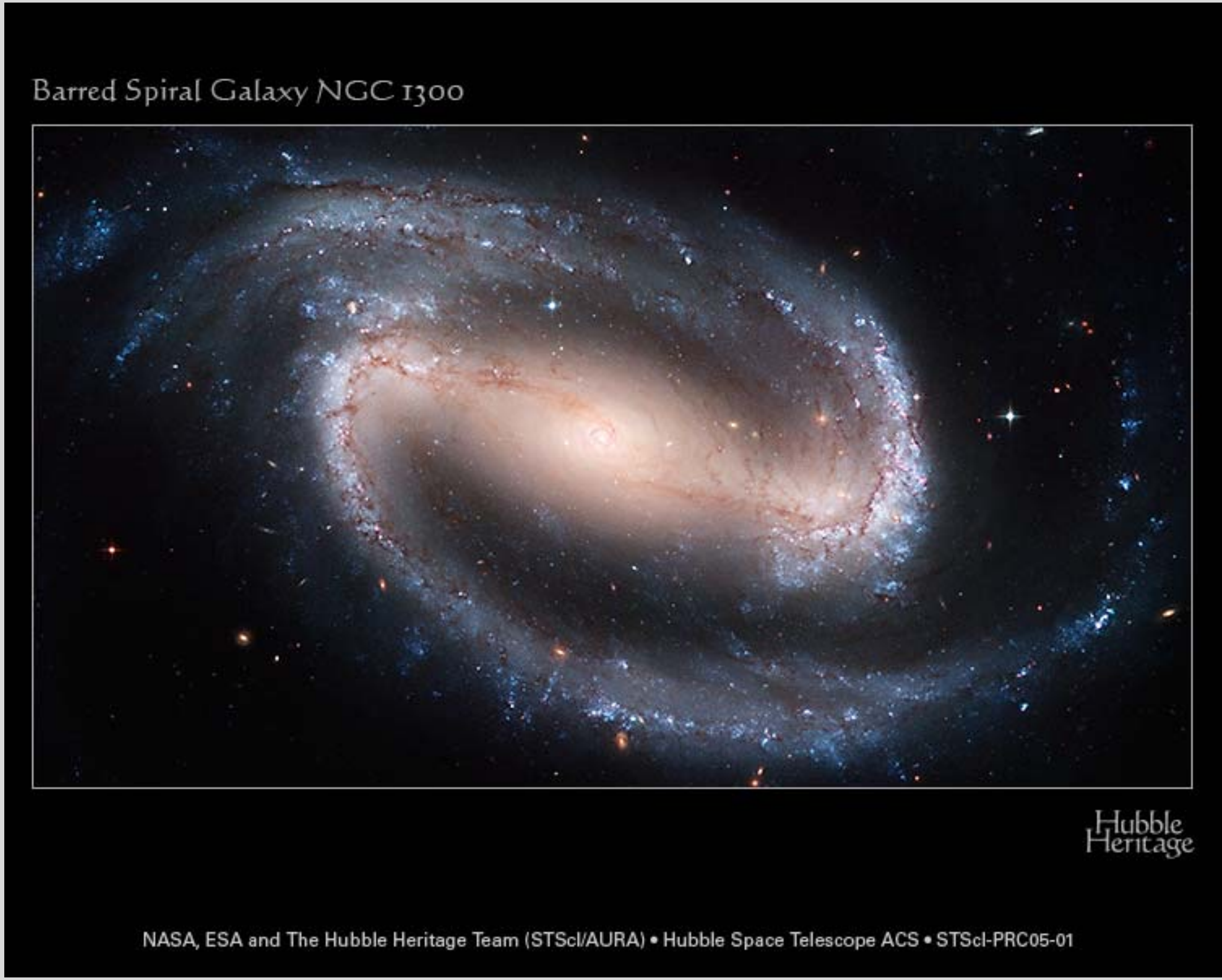
Grand design: 2 well-defined, symmetric spiral arms.

Flocculent: spiral arm “fragments”, not continuous

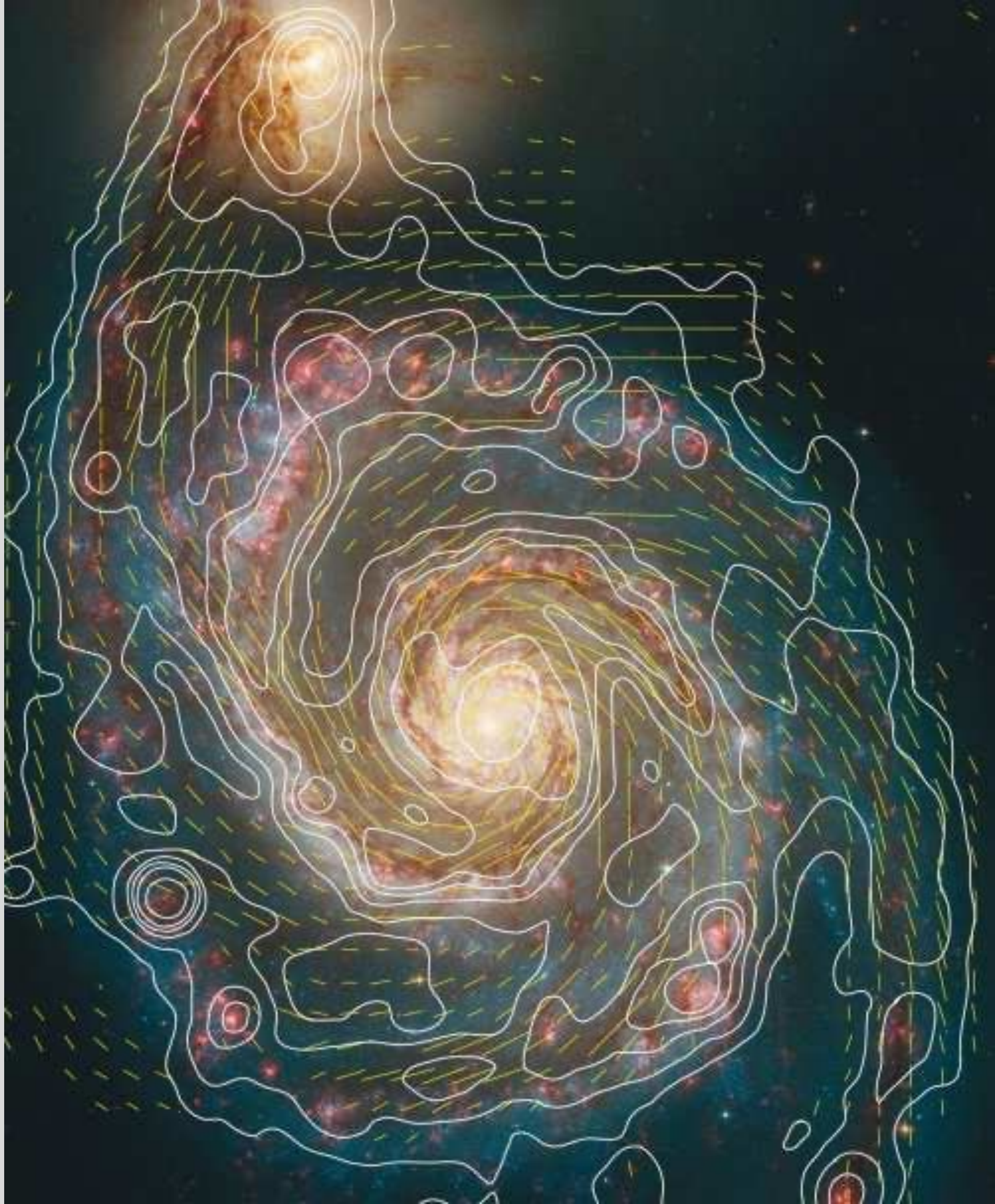
Multiple arms: 3, 4, etc

Barred spirals: arms coming off a central bar



Properties of Spirals

Very prominent at blue wavelengths, in Hα emission, and in radio continuum: star formation tracers.



Color image: optical/Hα
Countours: radio continuum

Properties of Spirals

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Strong dust lanes (often inside the arms): shows where gas enters the spiral arm.

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Velocity perturbations of ~ 20-30 km/s along spiral arms: arms are a significant enhancement of mass.

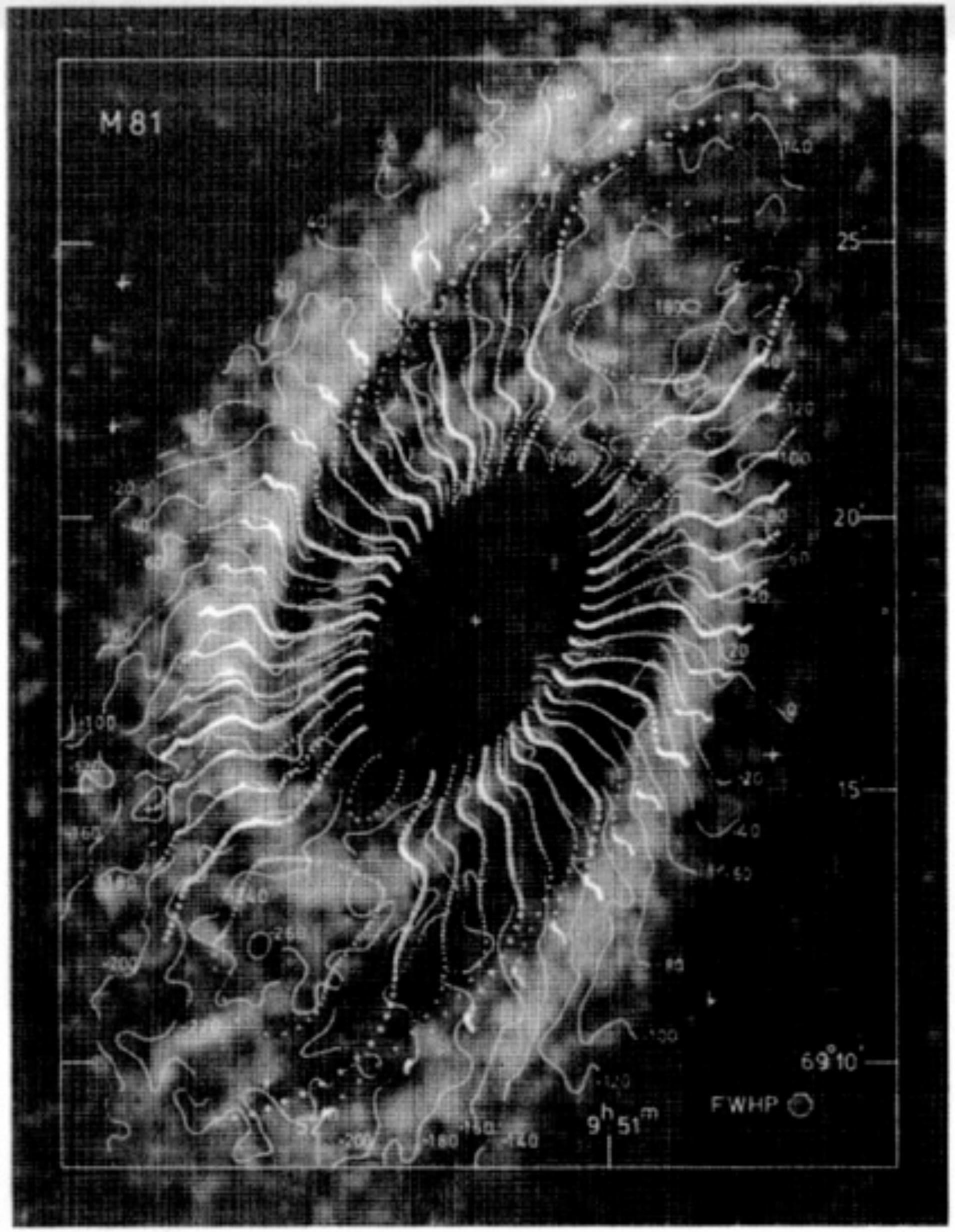

Figure 6-23. Constant-velocity contours of HI in the spiral galaxy M81. Solid lines represent observations made at the Westerbork Synthesis Radio Telescope, while chains of symbols represent predictions of a model based on the Lin-Shu hypothesis (Visser 1980). The contours are superimposed on an artificial photograph in which brightness is proportional to HI column density. The shaded circle at lower right represents the spatial resolution of the observations.

Properties of Spirals

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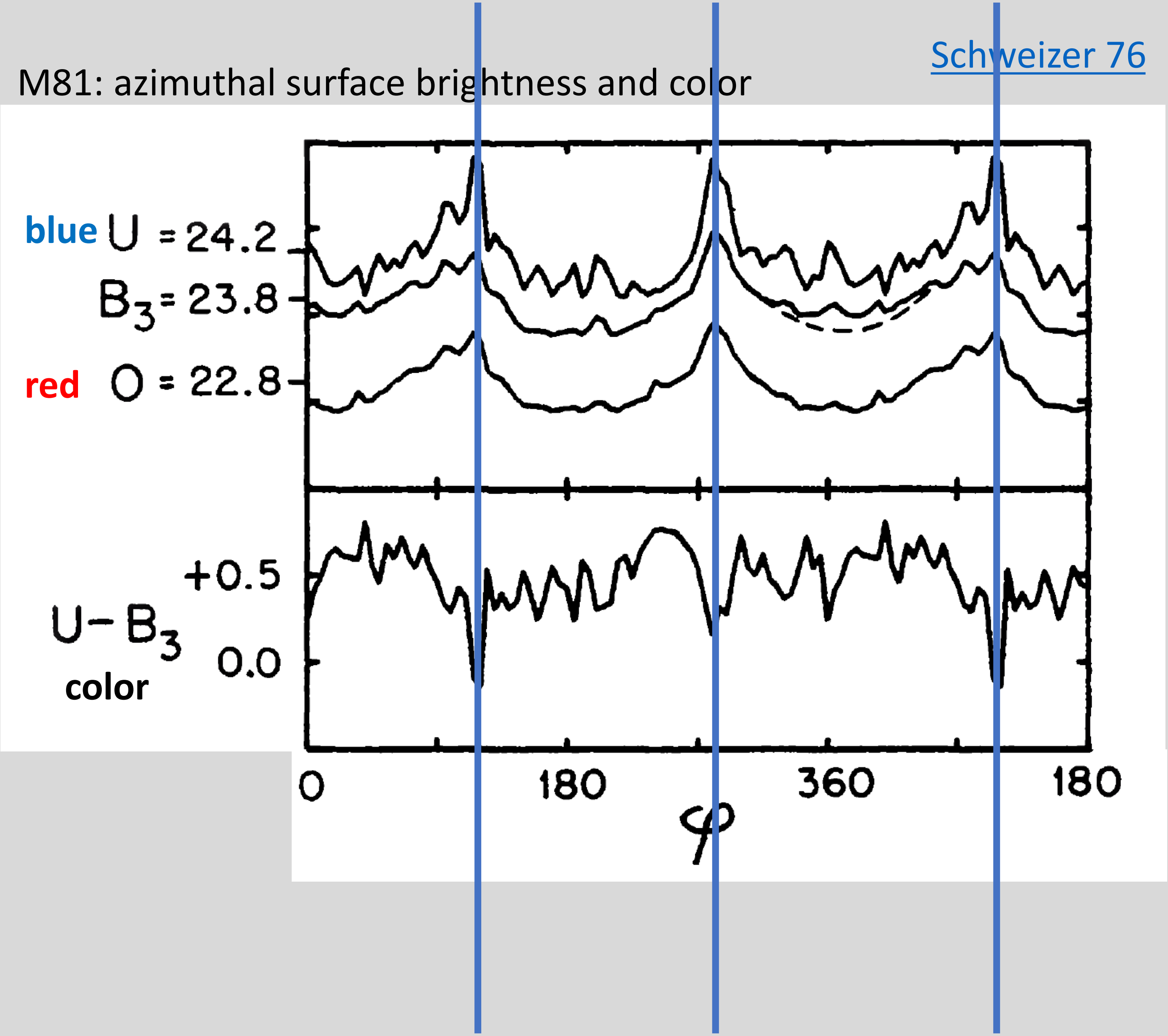
In red light, spiral arms are smoother, broader, lower in amplitude. Red light traces older stars, showing that the entire disk participates in the spiral structure.

Schweizer 76

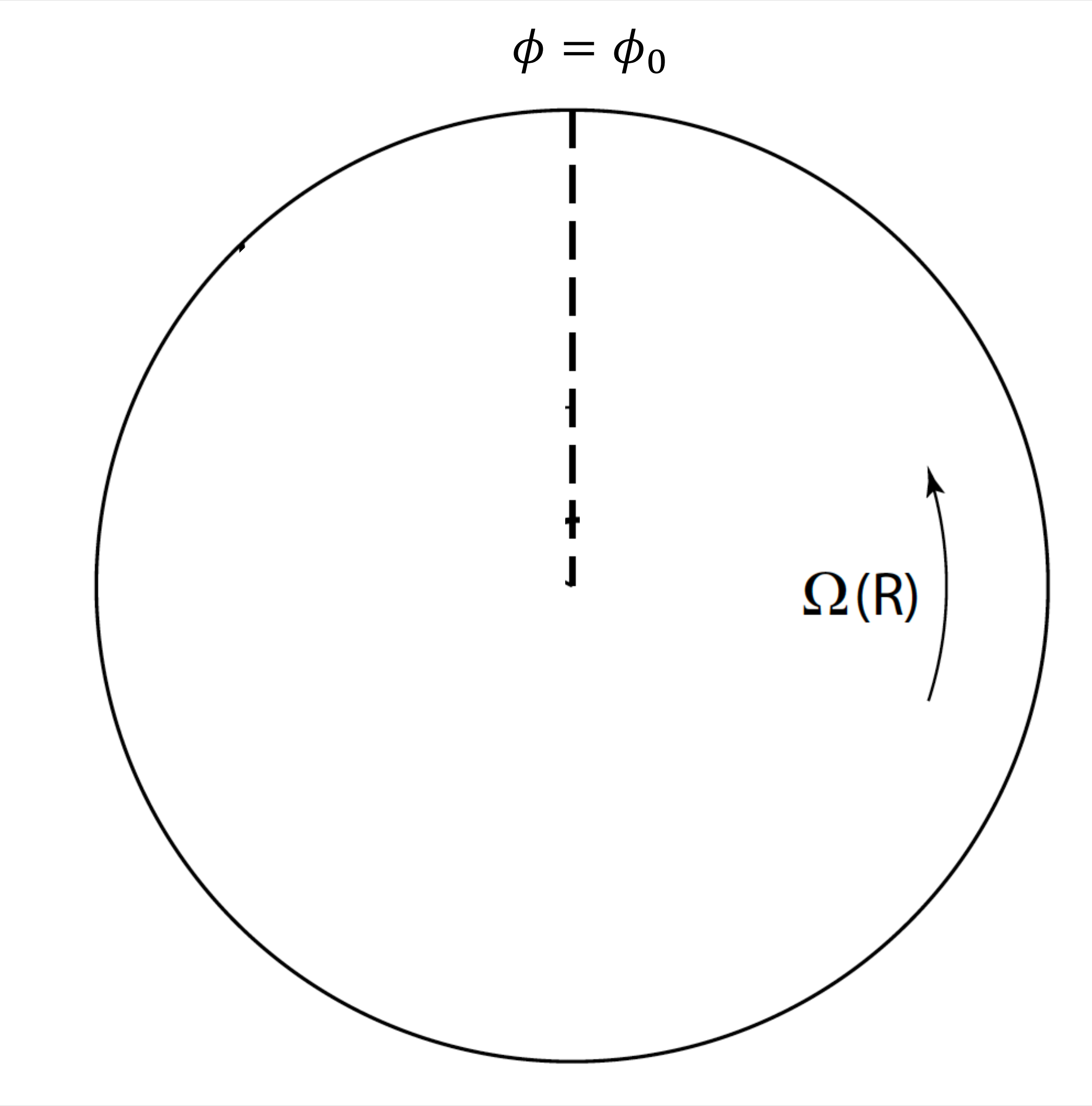
M81: azimuthal surface brightness and color



spiral arms

Winding Problem

Imagine painting a radial stripe on a rotating galaxy at some angle ϕ_0.



Winding Problem

Imagine painting a radial stripe on a rotating galaxy at some angle ϕ_0. After some time t, that stripe will "wind up" and follow the equation

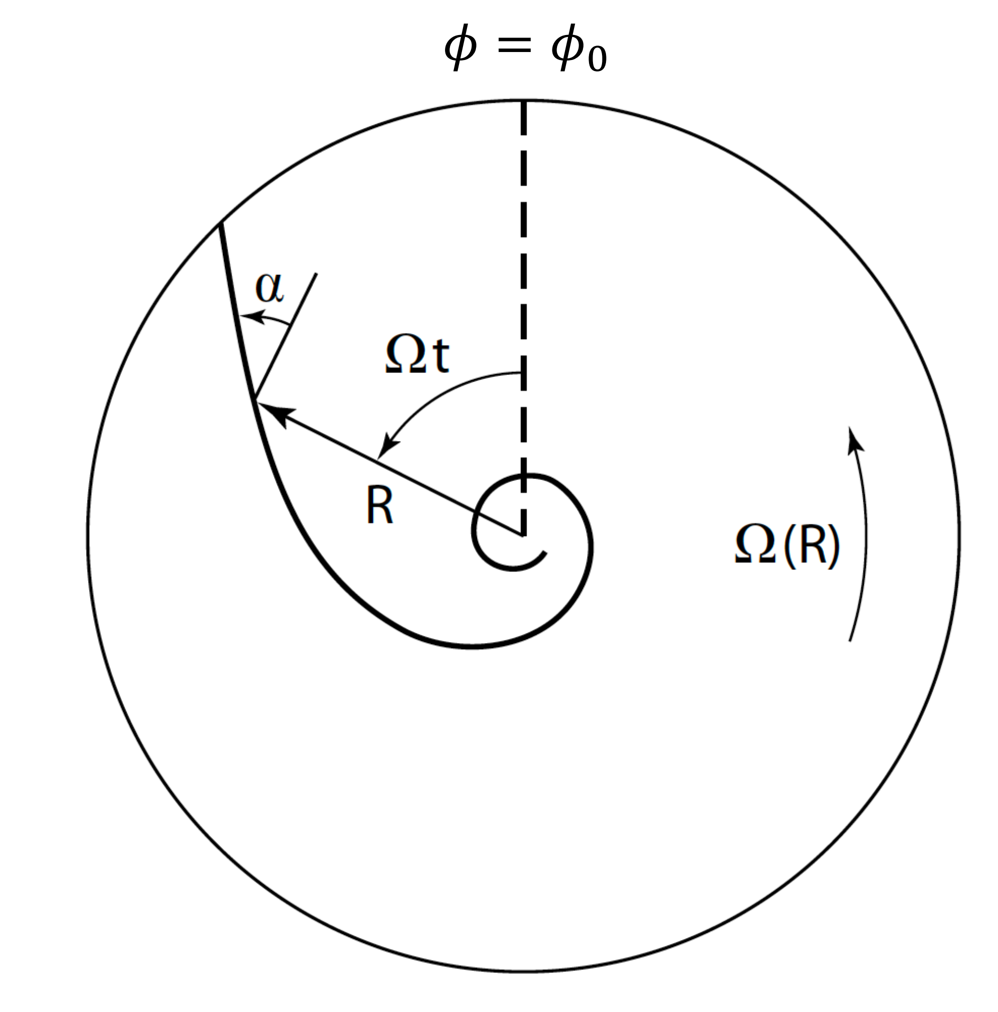
$$\phi(R,t) = \phi_0 + \Omega(R)t$$

where $\Omega(R) = V(R)/R$ is the angular rotation frequency.

The spiral has a pitch angle α defined by

$$\cot\alpha = \left|R\frac{\partial\phi}{dR}\right| = Rt\left|\frac{\partial\Omega}{\partial R}\right|$$

remember, here ϕ refers to the arm orientation, not the gravitational potential!



If we want the stripe to stay fixed in shape (but allow it to rotate), what is the requirement for $V(R)$?

We would need a constant $\Omega(R)$, meaning $V(R) \sim R$.

Do galaxies behave this way?

Winding Problem

How fast would galaxies "wind up"?

The stripe will wrap completely at a time t where

$$2\pi = |\Omega(R + \Delta R) - \Omega(R)| \times t$$

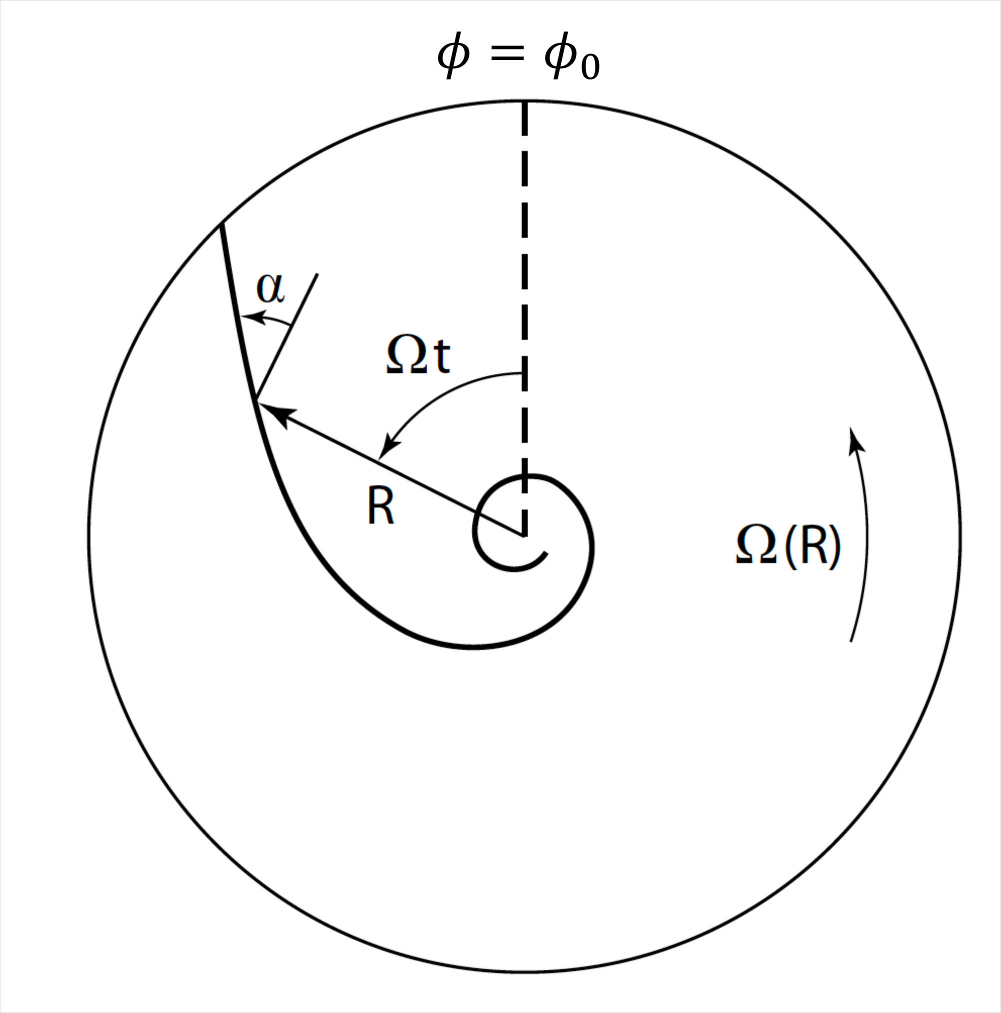
where ΔR would be the distance between wraps.

If $\Delta R \ll R,$ then $\Omega(R + \Delta R) = \Omega(R) + \frac{\partial \Omega}{\partial R} \Delta R$

$$\text{so} \quad 2\pi = \left| \frac{\partial \Omega}{\partial R} \Delta R \right| \times t$$

$$\text{or} \quad \Delta R = \frac{2\pi}{\left(\frac{\partial \Omega}{\partial R}\right) t} = \frac{2\pi R}{\cot \alpha}$$

where that last step comes from the definition of pitch angle.



Winding Problem

Put in some numbers. If

$$\Delta R = \frac{2\pi}{\left(\frac{\partial \Omega}{\partial R}\right) t} = \frac{2\pi R}{\cot \alpha}$$

Then for a Milky Way type galaxy with

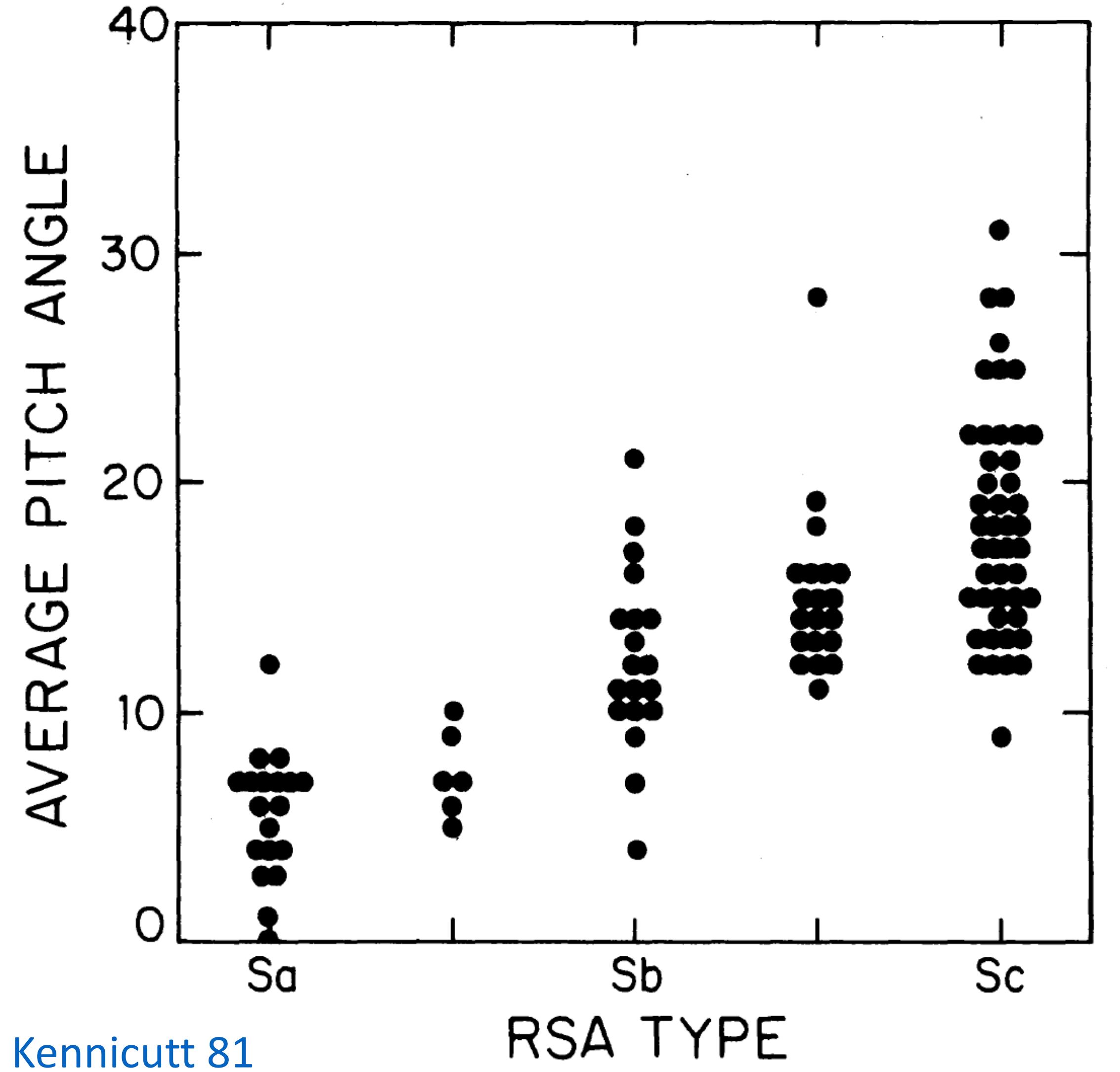
- $\Omega(R)R = V_c = 220$ km/s
- $R = 10$ kpc
- $t \approx 10$ Gyr

we get:

$\alpha = 0.25$ degrees
ΔR = 0.3 kpc

Hmm....

Look at observed pitch angles



Kennicutt 81