**In-plane Motion: 2D oscillations**

Now let's look at the 2D motion in the plane. We have $\ddot{x} = -\kappa^2 x$ which has some solution

*$\xi$ is just phase term, setting the starting point of the oscillation.*

$x = R - R_g$

$$x(t) = X\cos(\kappa t + \xi)$$

Look at azimuthal motion. Let $\psi$ be the angular coordinate along the orbit, so $\dot{\psi}$ is the angular velocity:

$$\dot{\psi} = \frac{L_z}{R^2} = \frac{L_z}{R_g^2}\left(1 + \frac{x}{R_g}\right)^{-2}$$

*Remember, $R_g$ is the radius of the circular orbit we are tweaking!*

where I've simply substituted in $R = R_g + x$ and then done some algebra.

If $x/R_g \ll 1$, I can do another expansion to get

$$\dot{\psi} \cong \Omega_g\left(1 - \frac{2x}{R_g}\right)$$

Now substitute x and be explicit about the derivative

$$\dot{\psi} = \frac{\partial\psi}{\partial t} = \Omega_g\left(1 - \frac{2X}{R_g}\cos(\kappa t + \xi)\right)$$

And now integrate

*$\psi_0$ is just another phase term that sets the starting point of the orbit.*

$$\psi(t) = \Omega_g t - \frac{2\Omega_g X}{\kappa R_g}\sin(\kappa t + \xi) + \psi_0$$

**Finally: Epicycles**

Let's put an (x,y) cartesian coordinate system centered on $(R_g, \Omega_g t + \psi_0)$.

Since $\psi(t) = \Omega_g t - \frac{2\Omega_g X}{\kappa R_g}\sin(\kappa t + \xi) + \psi_0$ we have

$$\left.\begin{aligned} x(t) &= \quad X\cos(\kappa t + \xi) \\ y(t) &= -Y\sin(\kappa t + \xi), \text{ where } Y \equiv \frac{2\Omega_g X}{\kappa R_g} \end{aligned}\right\}$$

*This is simply the equation of an ellipse!*

The star moves on an ellipse around $R_g$, as $R_g$ moves around the galaxy on a circular orbit. The motion is described as an **epicycle** with a **guiding center** $R_g$! The frequency $\kappa$ is called the **epicyclic frequency**.

Notes:

- The ellipse has an axis ratio of $X/Y = \kappa/(2\Omega_g)$
- For typical galactic potentials $Y > X$, so the ellipse is elongated tangentially
- Epicycles are retrograde. *Why?*
  - **Conservation of angular momentum.**
  - *When the star is further out from the guiding center it moves more slowly and lags the guiding center.*
  - *When the star is closer in, it moves more quickly and leads the guiding center.*

## Epicycles around a point source

Think of Keplerian motion: $V_C \sim R^{-0.5}$, $\Omega = V_c/R \sim R^{-1.5}$,

Epicyclic frequency: $\kappa^2 = R\frac{\partial \Omega^2}{\partial R} + 4\Omega^2 = \mathrm{R}\frac{\partial (R^{-3})}{\partial R} + 4\Omega^2 = \mathrm{R}(-3R^{-4}) + 4R^{-3} = R^{-3}$

So: $\kappa \sim R^{-1.5} \sim \Omega$, and the ratio of the ellipse is $\frac{X}{Y} = \frac{\kappa}{2\Omega_g} = \frac{1}{2}$

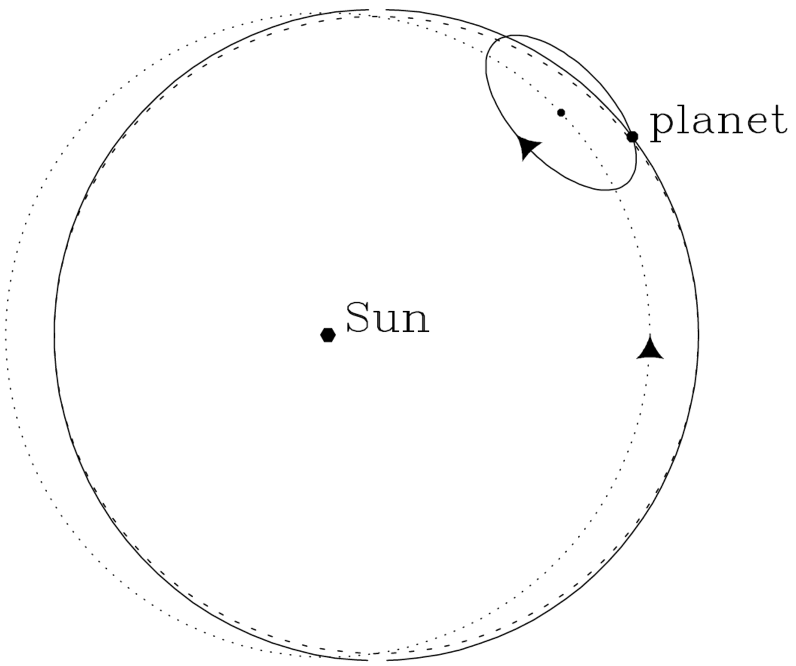
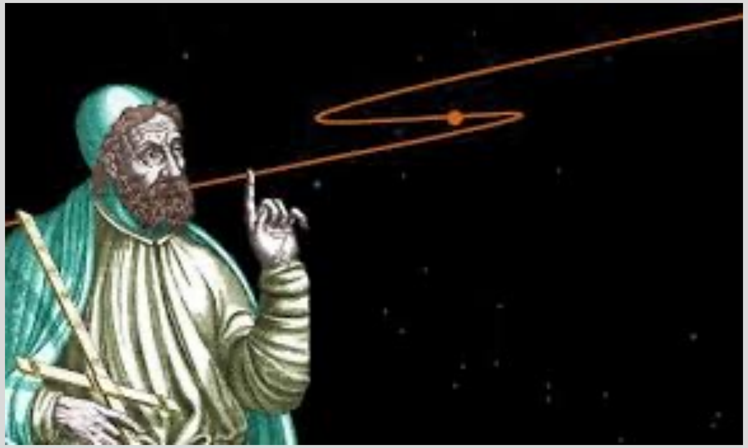



**Figure 3.7** An elliptical Kepler orbit (dashed curve) is well approximated by the superposition of motion at angular frequency $\kappa$ around a small ellipse with axis ratio $\frac{1}{2}$, and motion of the ellipse's center in the opposite sense at angular frequency $\Omega$ around a circle (dotted curve).

*What did Ptolemy get wrong?*



**Clarification on the epicyclic approximation:**

The epicycle:

- It is an epicyclic loop only in the rotating frame of reference.

- In any reasonable potential, the epicyclic frequency ($\kappa$) is comparable to the orbital frequency ($\Omega$) to within a factor of a few.

- So orbits do not gyrate wildly, they just deviate slightly from circular.

Why is the epicycle retrograde compared to the guiding center mortion?

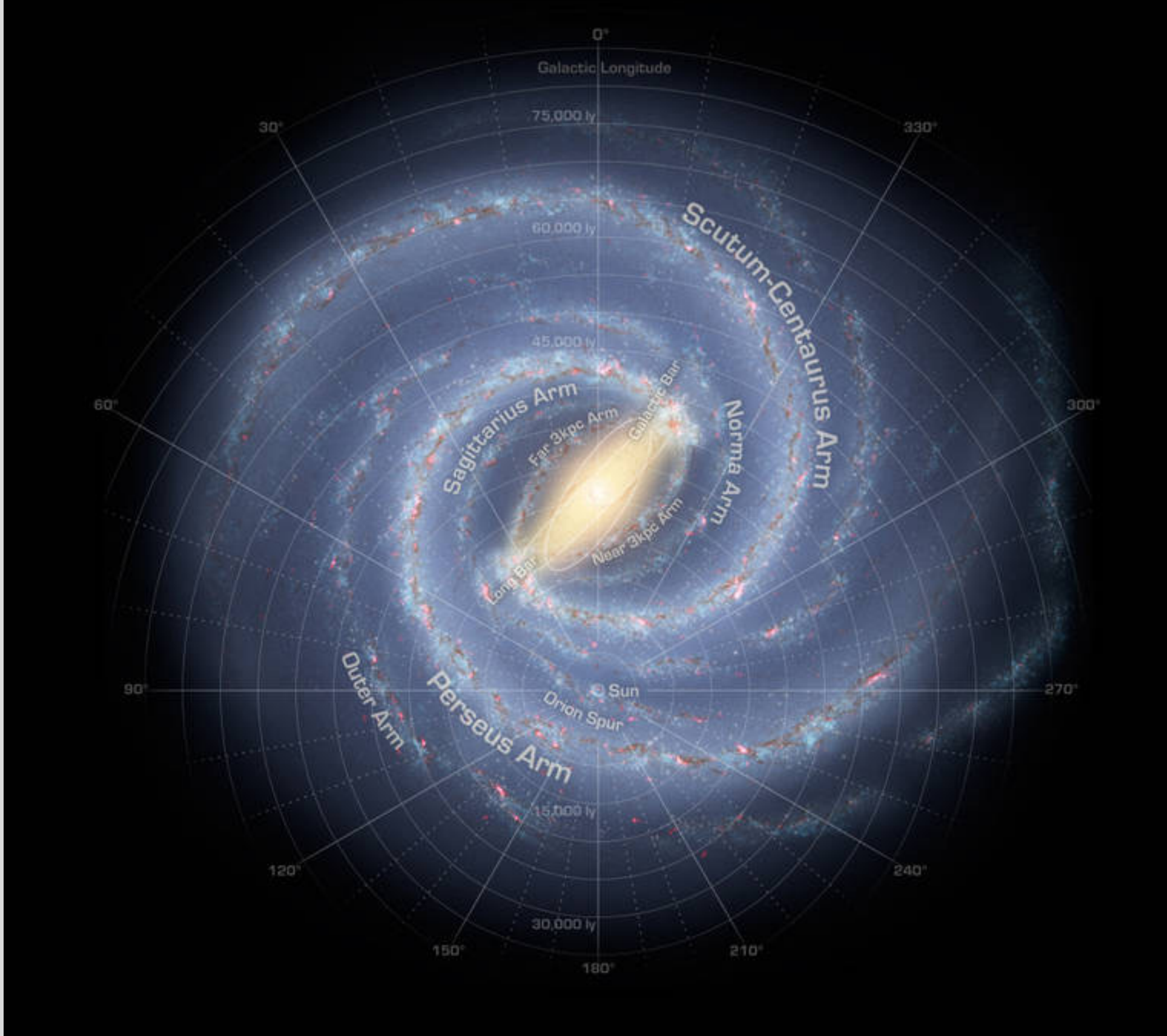
- Momentum ($L = rv_\theta$) is conserved on the orbit.
- When the star is inside $R_g$ it has a higher angular velocity, so it moves ahead of the guiding center.
- As it moves ahead, it is moving faster than circular, so it also drifts outwards.
- As it drifts outwards it also slows down in $v_\theta$ to keep angular momentum conserved.
- As it moves beyond $R_g$ and slows down in $v_\theta$ it lags the guiding center.
- Since it now is moving slower than circular, it starts to drift back inwards.
- and the cycle repeats, with the epicycle being retrograde compare to the guiding center motion.

**Spiral Galaxies**

From our study of the Milky Way:

- Rotating disk
- Exponential in R an z
- Thick and thin disks
- Spiral arms and bar
- Stellar populations:
  - Star Formation
  - Chemical Evolution
- Stellar halo

What about other spiral galaxies? How can lessons learned in the Milky Way be generalized to spiral galaxies as a whole?



## Spiral Galaxies: Structure

Spiral galaxy luminosity profiles are reasonably well described by a varying combination of disk and bulge profiles.

**Disk:** linear surface brightness profile (mag arcsec$^{-2}$), so exponential fall-off with radius. Characterized by:

- central surface brightness: $\mu_0$
- radial scale length: $h_R$

**Central bulge:** profile varied

- classical $r^{-1/4}$ profile (early mergers?)
- exponential profile ("pseudo-bulges", maybe from scattering of disk stars?)

Disks also typically have **stellar halos**, but these are too diffuse to be observed in integrated light.

NGC 7331, Hager & Grossman

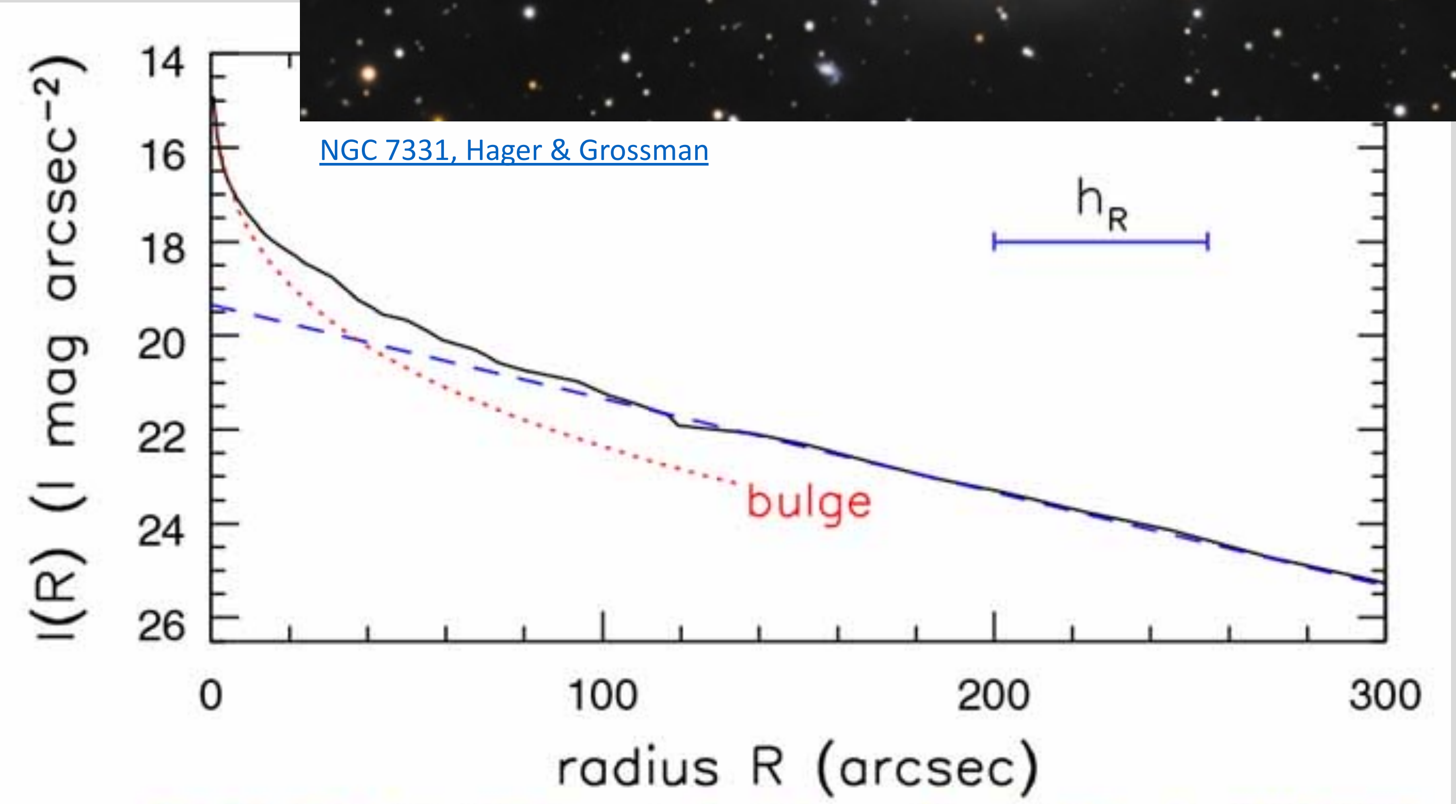



Fig 5.4 (R. Peletier) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007

## Spiral Galaxies: Structure

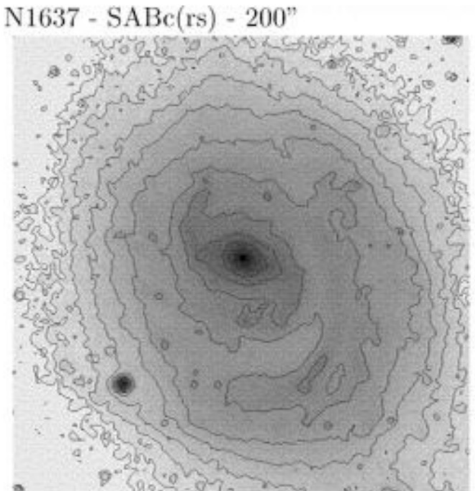
But spiral galaxies are not perfectly symmetric. Fit ellipses to isophotes. If the disk is axisymmetric (but inclined), ellipses should be constant with radius.

Changing isophotes ⇒ asymmetry: best fit ellipse as a function of radius can change orientation ("position angle" or PA), ellipticity, and deviation from ellipse (A4).

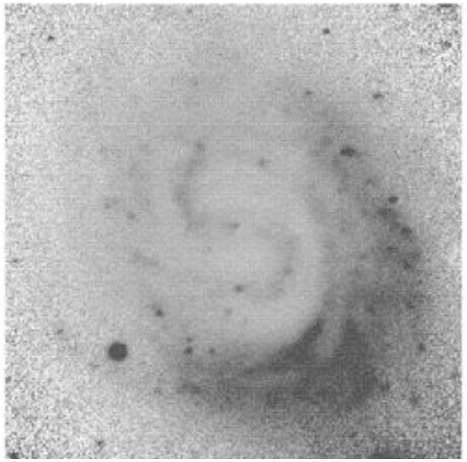
"Fourier modes" (m): At some radius, think of galaxy surface brightness as a function of angle. If the disk is axisymmetric, this would be a constant.

- m=0: constant, axisymmetric
- m=1: one hump, lopsided
- m=2: two humps, bar or grand design spiral arms
- m=3+: multi-armed spirals, other weirdness

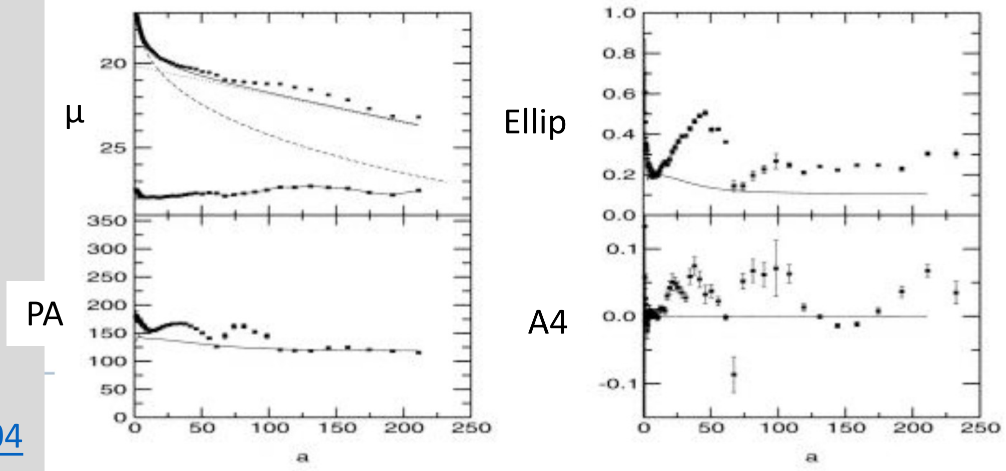
Many different metrics exist to quantify asymmetry.




Raw image



After axisymmetric model subtraction



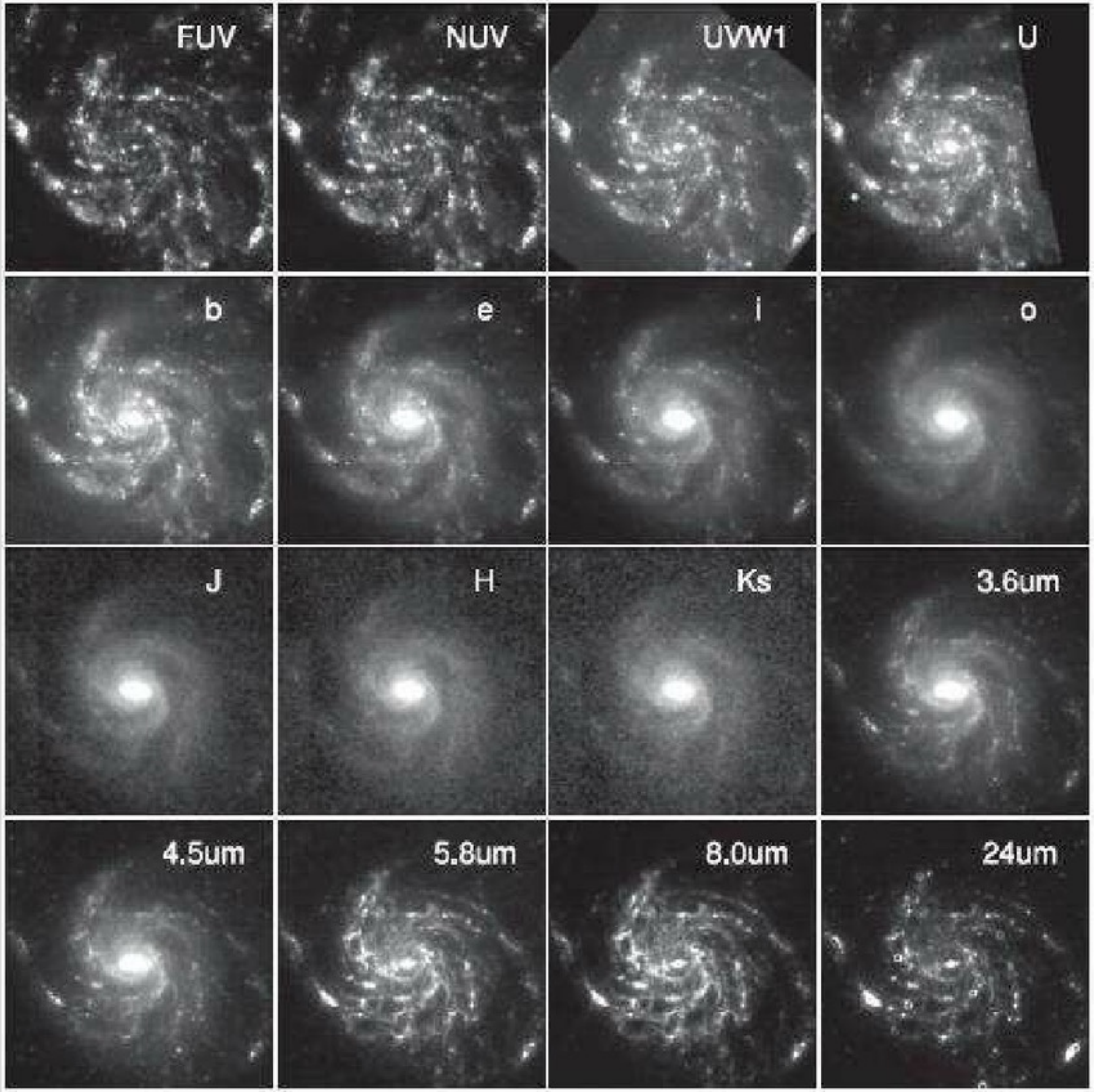

de Souza+ 04

Morphology depends on wavelength!

UV/blue: see young stars, spiral arms enhanced.

red/near-IR: dominated by light from old stars. Smoother, emphasizes central bulge

mid-IR: see emission from dust heated by young stars, traces spiral arms.



## Spiral Galaxies: Vertical Structure

Many disk galaxies observed to have a thick disk, but not all.

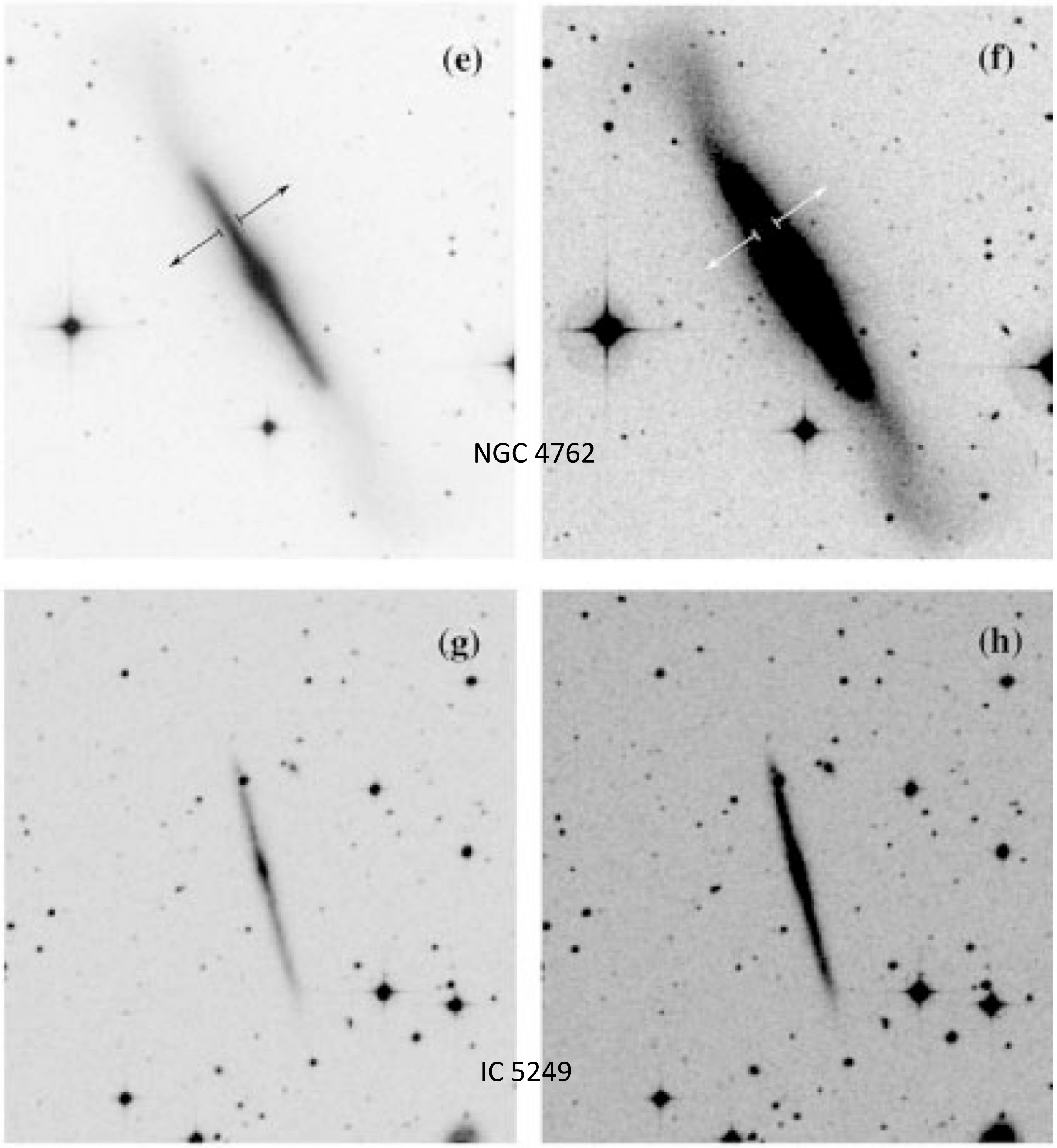
Disks also are seen to flare or warp in the outskirts.

Why might this be?

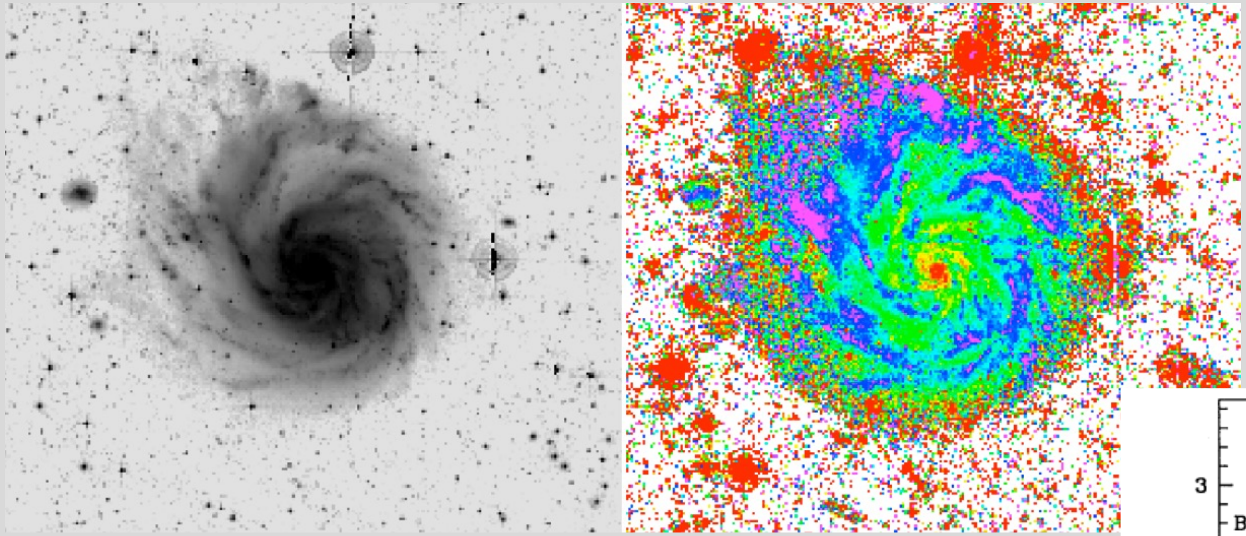
- Interactions?
- Low density in outskirts: no restoring force for vertical oscillations

$$\sigma_z^2 \approx 2\pi G \Sigma h_z$$

from Freeman 02 ARAA



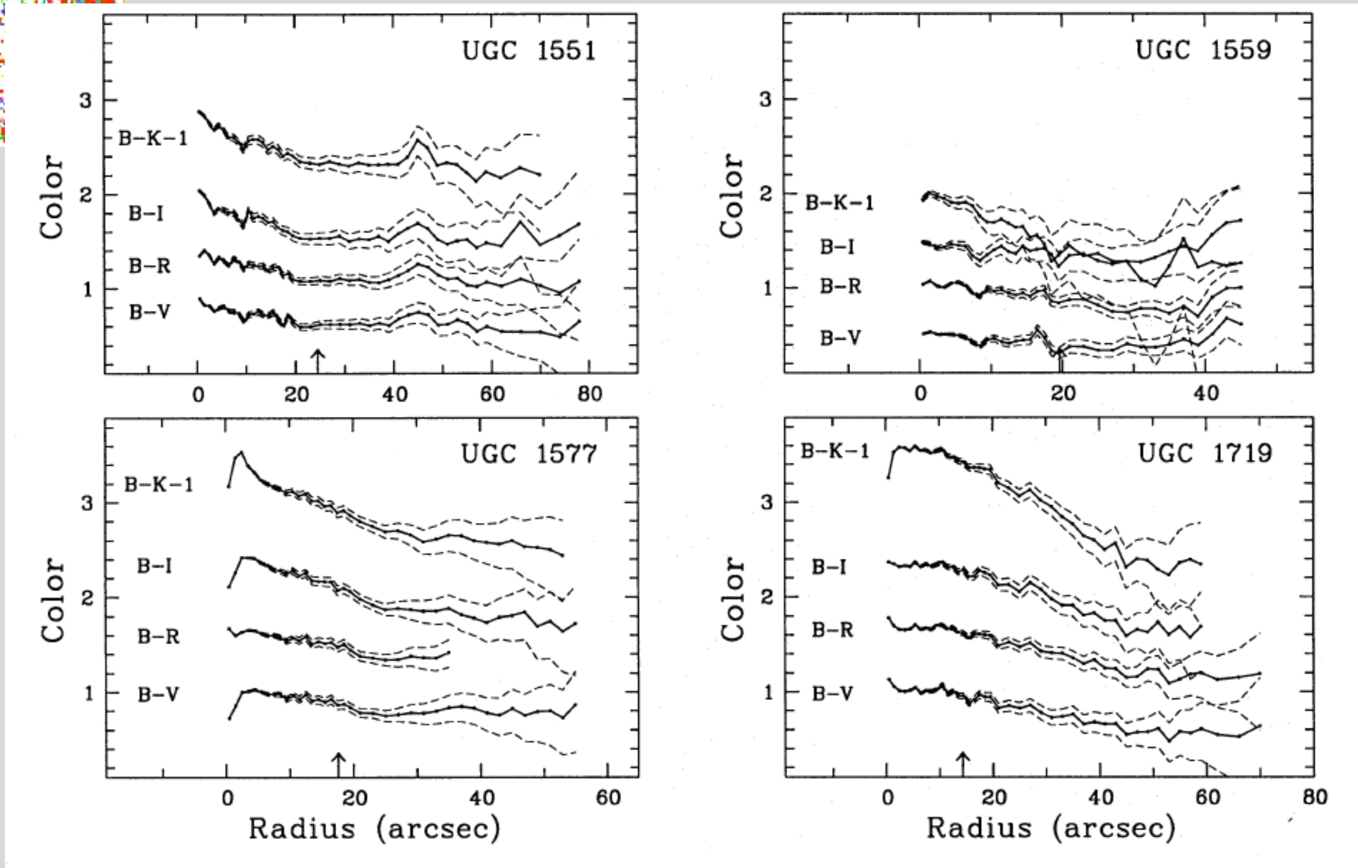
# Spiral Galaxies: Color Profiles



M101 color gradient
(false color, but maps real color)

Mihos+ 13

Color gradients in other spirals
de Jong 1996




Disks typically show color gradients:

- Galaxy colors become bluer in the outskirts
- Fitted scale lengths ($h_R$) are shorter in the red.

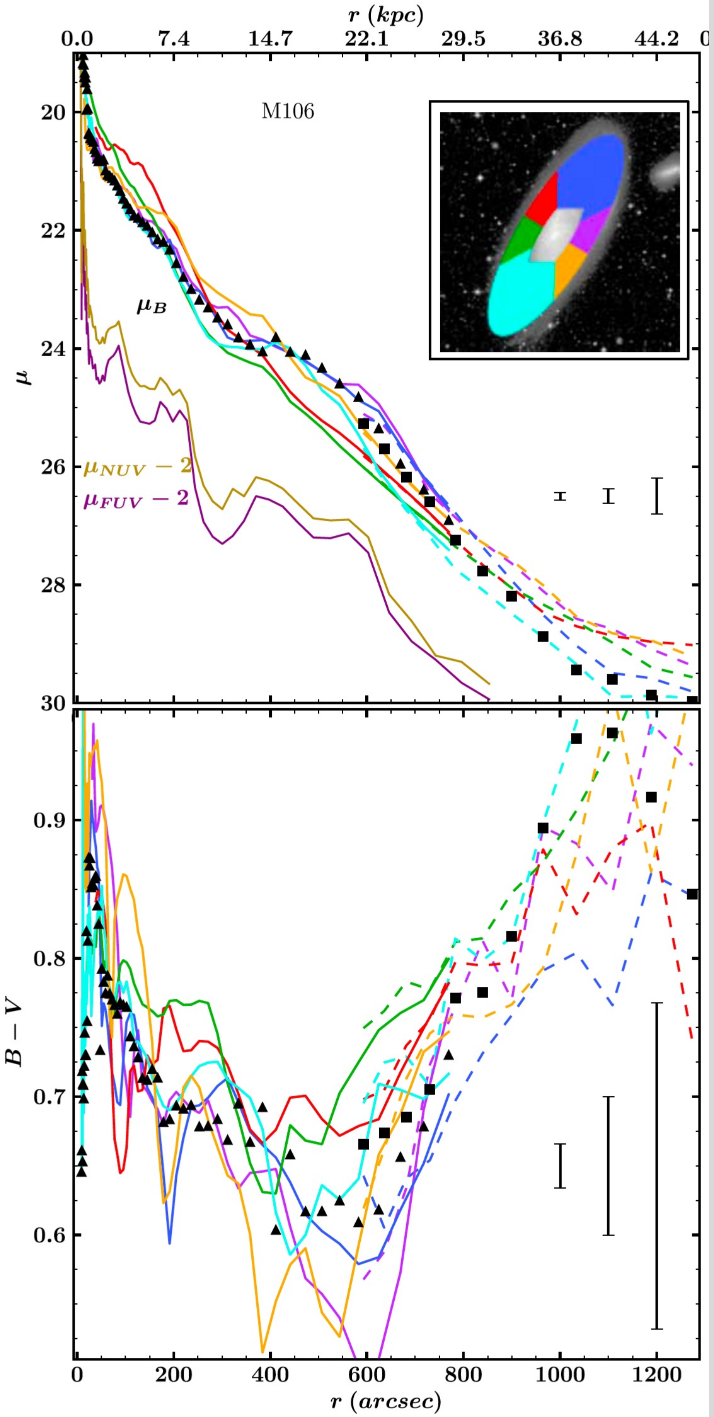
## Spiral Galaxies: Color Profiles

But sometimes color gradients reverse in the extreme outskirts.

M106

Surface Brightness

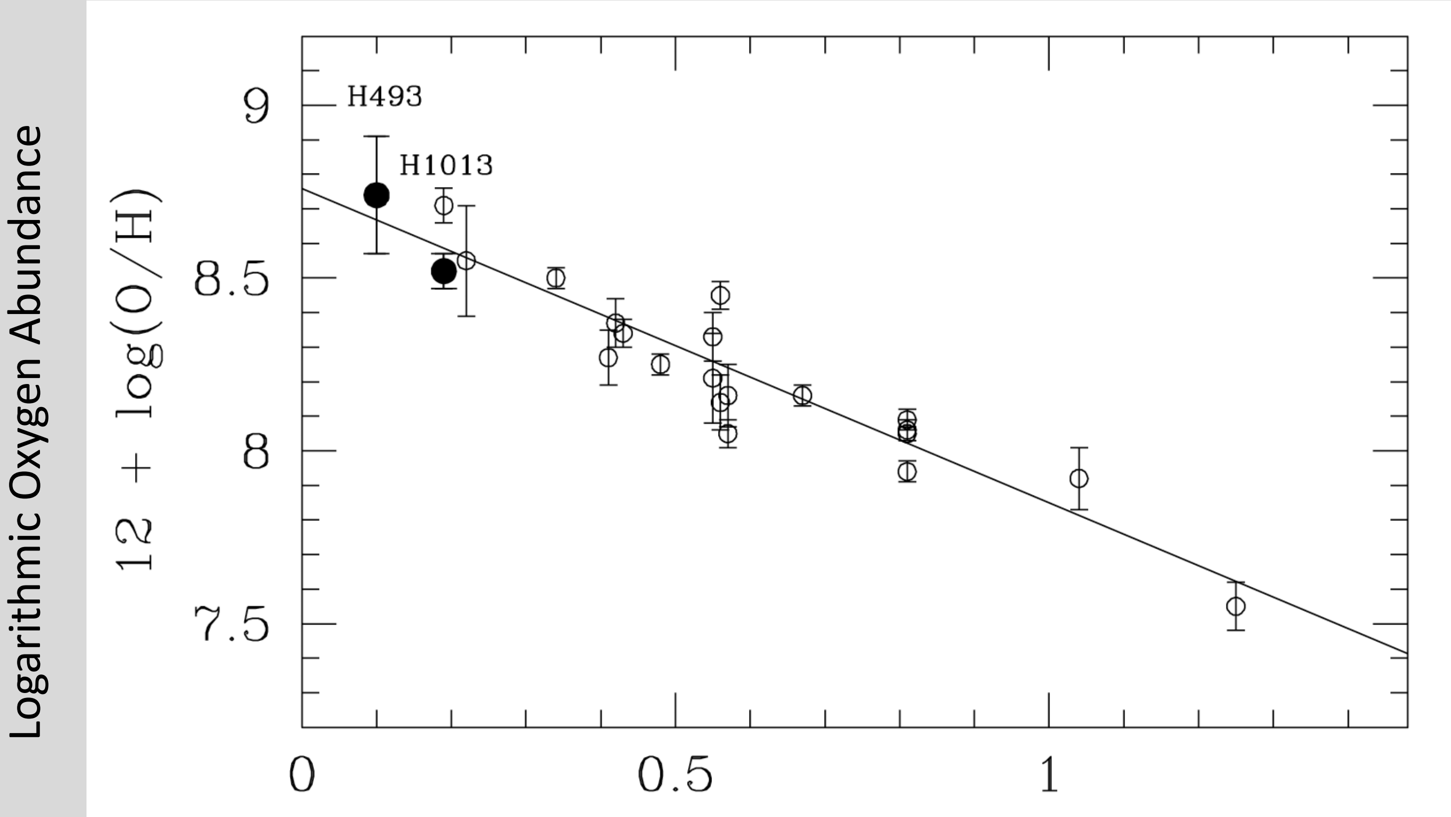
Color




Watkins+ 16

## Spiral Galaxies: Metallicity Gradients

Spiral galaxies typically also show metallicity gradients: outskirts are more metal poor than inner regions.

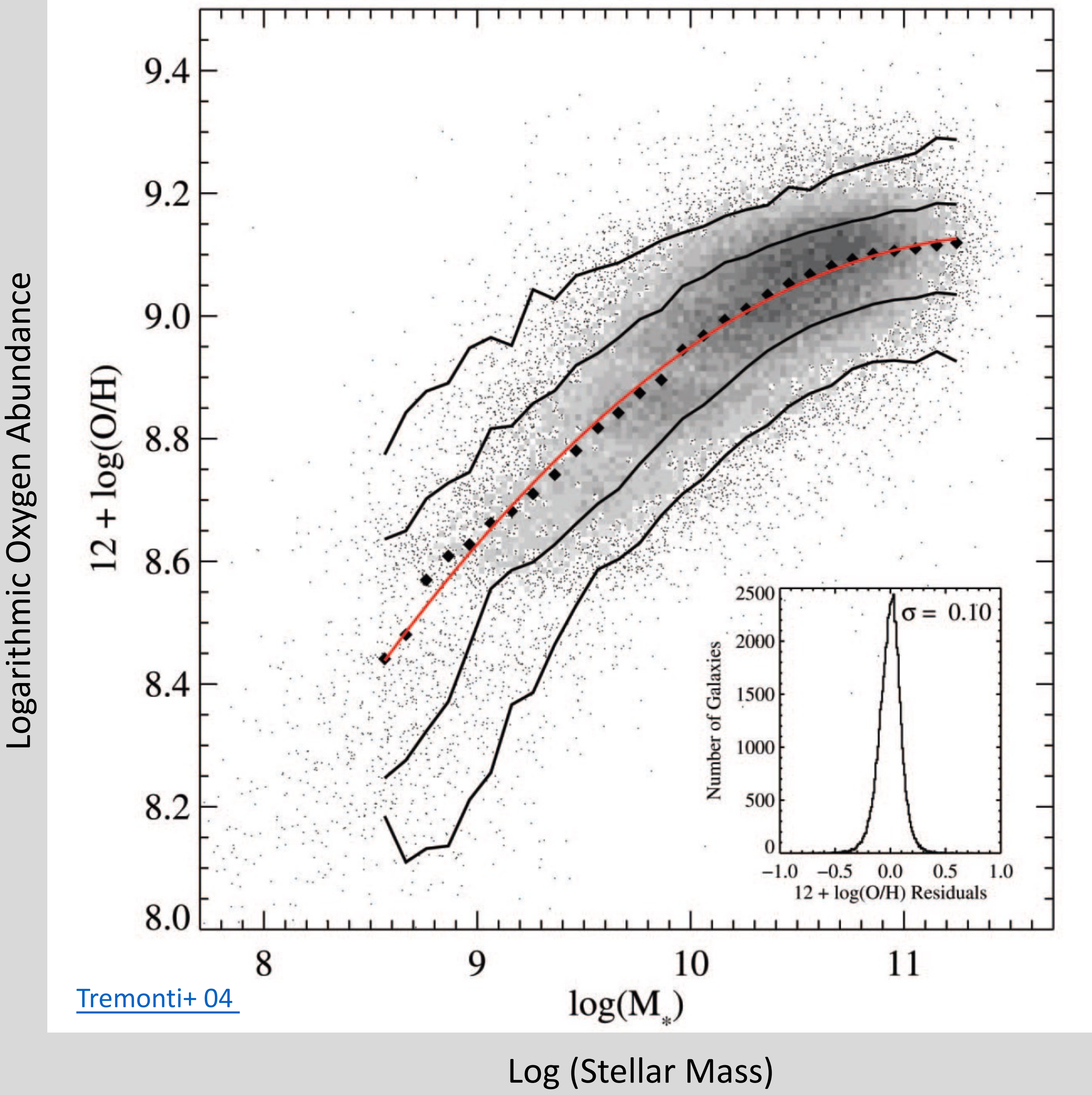



Bresolin 07

**Spiral Galaxies: Metallicity**

Low mass galaxies are typically also metal-poor.

"Mass-metallicity relationship", also seen in elliptical galaxies.




Tremonti+ 04

## Spiral Galaxies: Atomic/Molecular Gas

Spirals are rich in cold atomic and molecular gas – fuel for star formation

- Atomic: probe via 21cm neutral hydrogen line
- Molecular: no $H_2$ emission, so trace via other molecules (CO, etc)

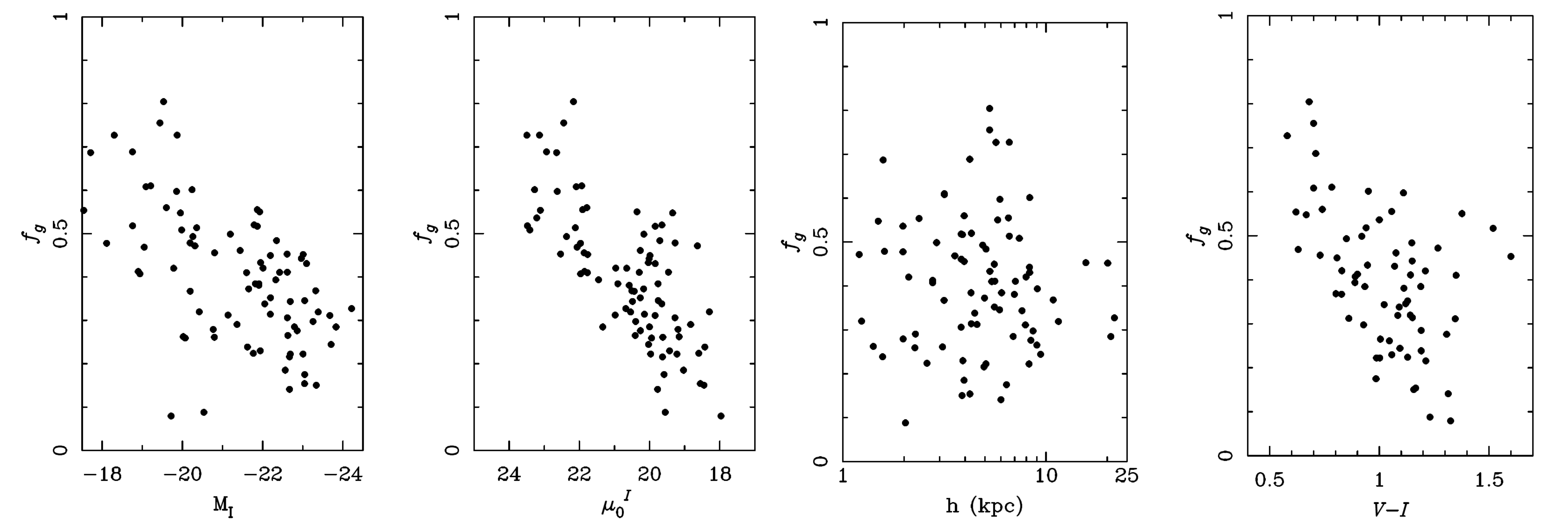
$$M_{gas} = M_{HI} + X \times I_{CO}$$

where

$M_{HI}$ = HI mass
$I_{CO}$ = CO line flux
X = scale factor (uncertainty!)

"gas fraction": $f_g = \frac{M_{gas}}{M_{gas}+M_{stars}}$

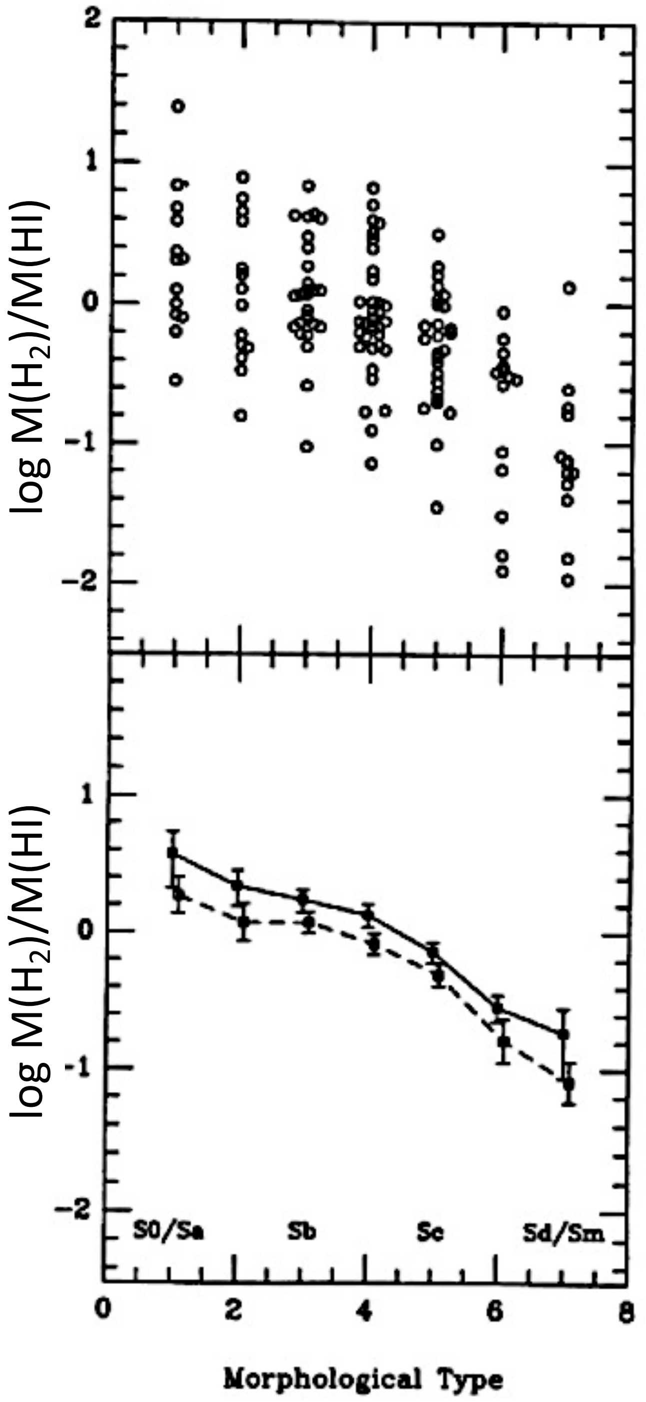
McGaugh & de Blok 97




luminosity → surface brightness → size → (blue) color (red)

**Spiral Galaxies: Atomic/Molecular Gas**

Late type spirals are HI-rich but $H_2$-deficient

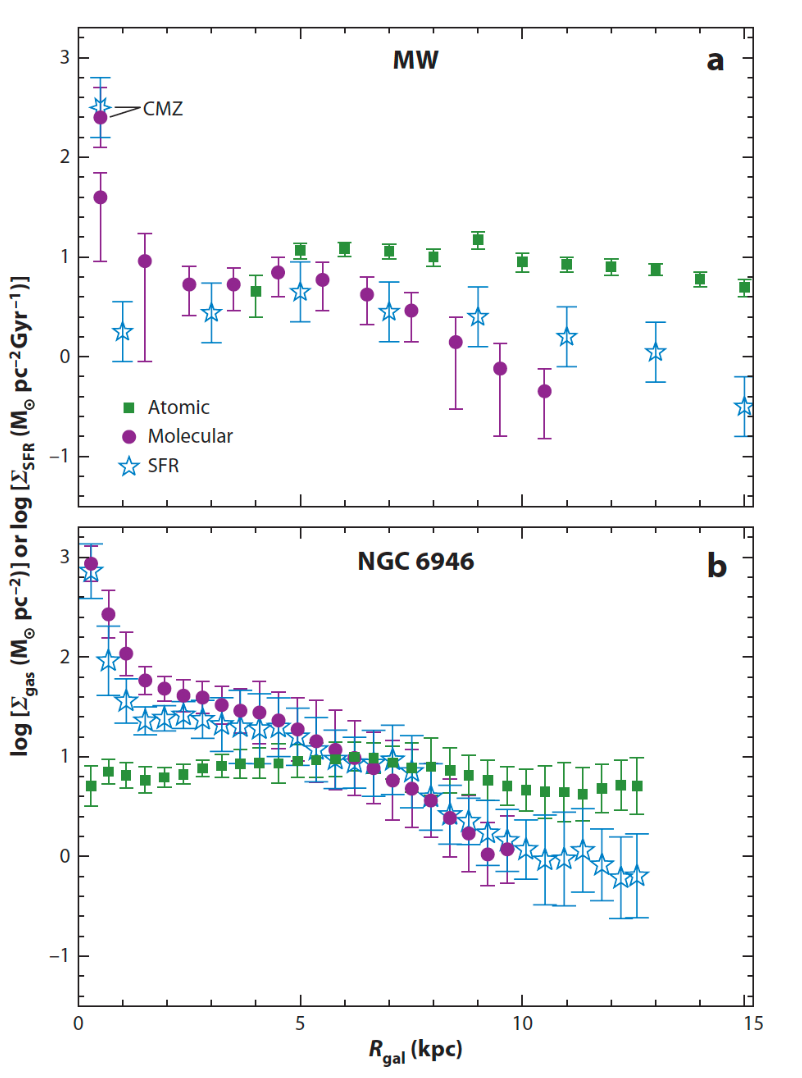


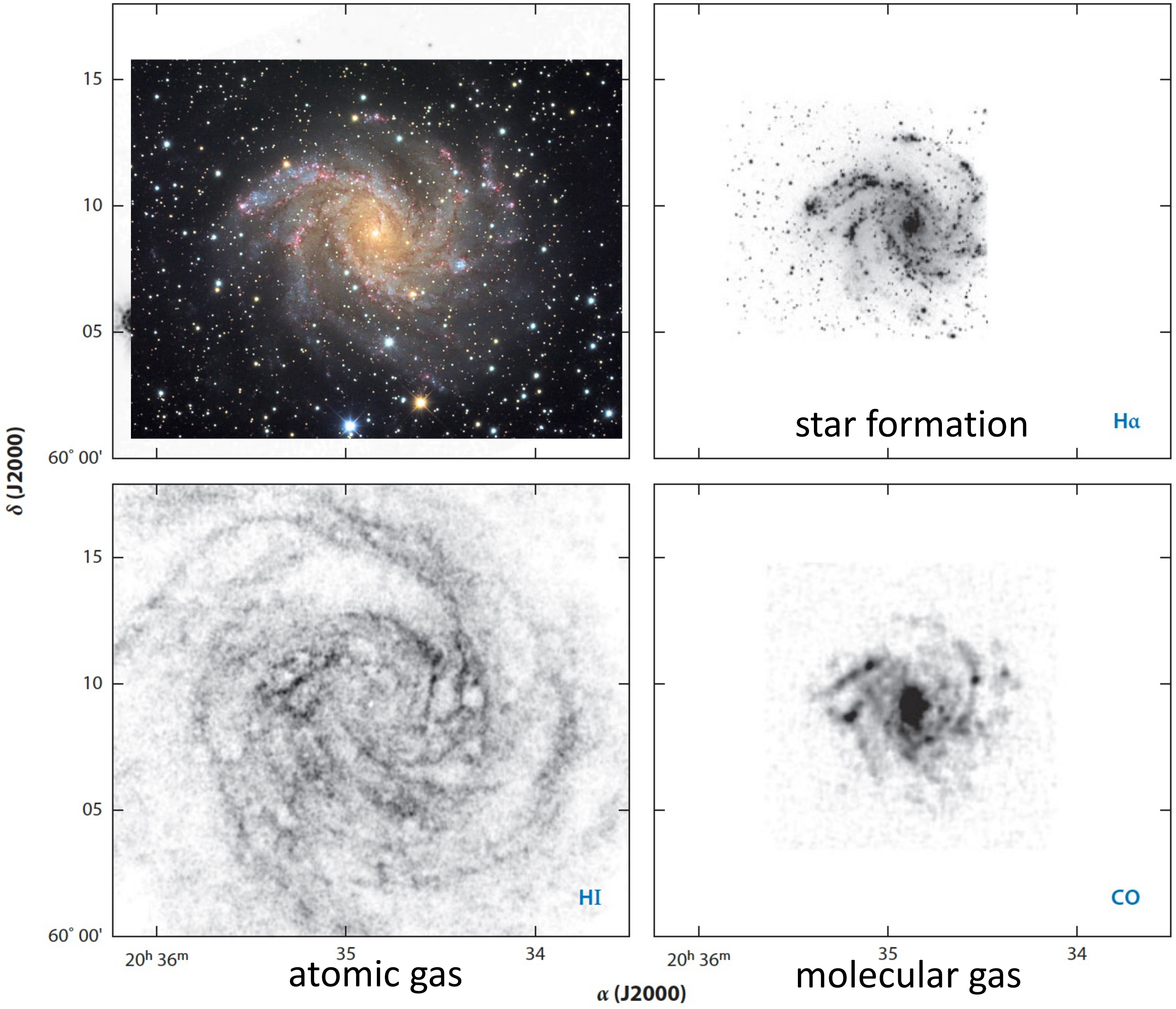

Young & Knezek 89

## Spiral Galaxies: Atomic/Molecular Gas

HI is also typically much more radially extended than the molecular gas

log (surface density)




Kennicutt & Evans ARAA 2012

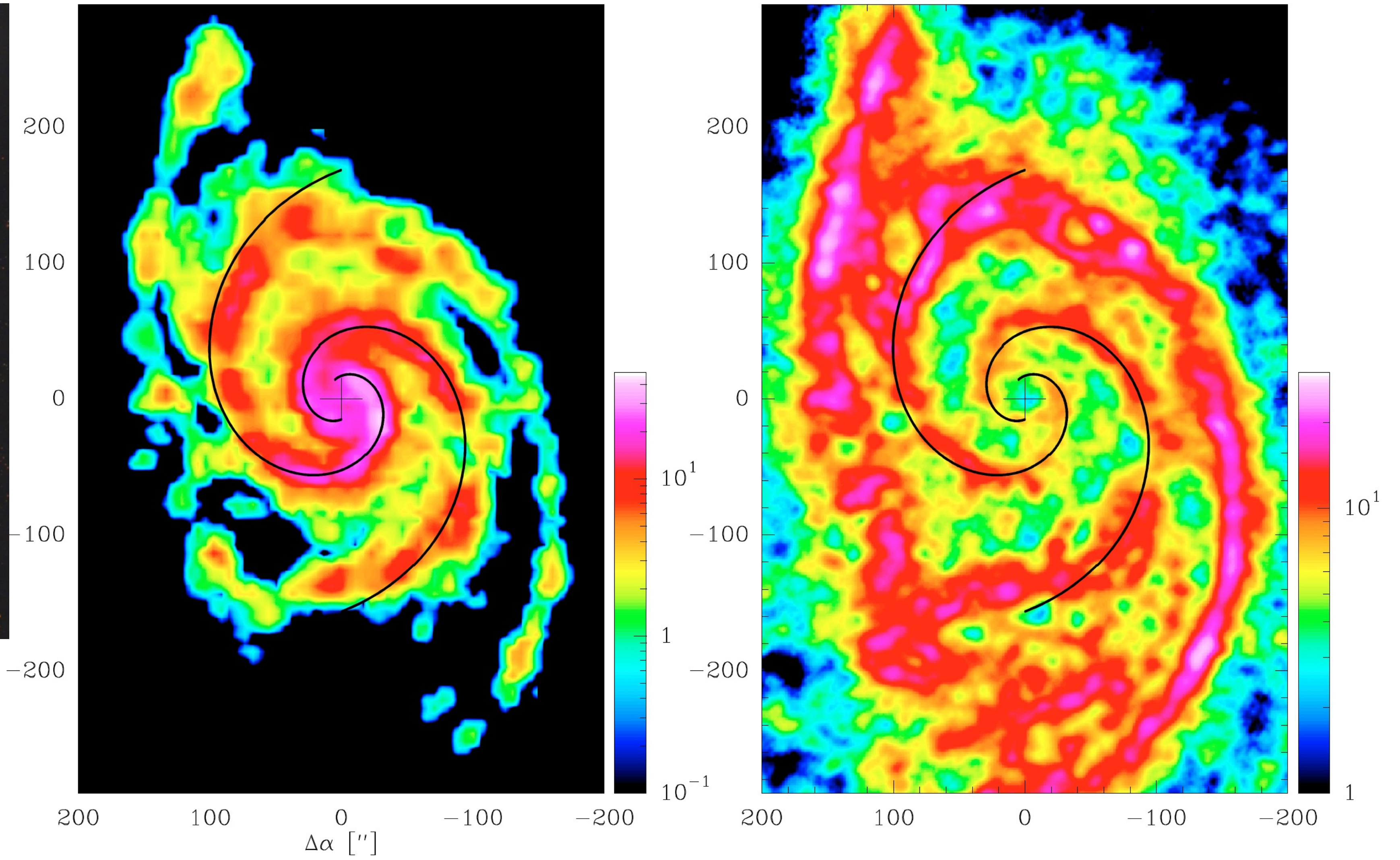
**Figure 6**

High-resolution maps of star-formation tracers and cold-gas components in NGC 6946. (*Top left*) *Spitzer* 24-μm dust emission from the SINGS/KINGFISH Project (Kennicutt et al. 2011), (*top right*) Hα emission from Knapen et al. (2004), (*bottom left*) HI emission from the VLA THINGS Survey (Walter et al. 2008), and (*bottom right*) CO $J = 2 \rightarrow 1$ emission from the HERACLES Survey (Walter et al. 2008). Note that the HI map extends over a much wider area than the CO, Hα, and 24-μm observations.

# Spiral Galaxies: Atomic/Molecular Gas

Molecular Gas

Atomic Hydrogen




Hitschfeld+ 09

**Fig. 1.** *Left*: map of integrated CO 2–1 intensities on the antenna temperature scale in K km s$^{-1}$ at 11″ resolution (cf. Paper I). A logarithmic spiral is shown in all figures of M 51 to guide the eye. *Right*: map of H I column densities (Walter et al. 2008) in units of $10^{20}$ cm$^{-2}$ at a resolution of ~6″.

**Recapping…**

**Disk galaxies show trends**

Low luminosity / low surface brightness galaxies: lower in metallicity, bluer, more gas-rich, and their gas is mostly atomic.

High luminosity / high surface brightness galaxies: higher in metallicity, redder, lower gas fractions, more of their gas is in molecular phase.

**Disk galaxies show gradients**

Outer regions are lower in metallicity, bluer, more gas-rich, and atomic gas dominated.

Inner parts are higher in metallicty, redder, less gas-rich, and more molecular gas.

*Note: many exceptions exist, enough so that there are whole classes of galaxies that defy these relationships. But as a general thumbnail sketch of galaxies, this is reasonable.*