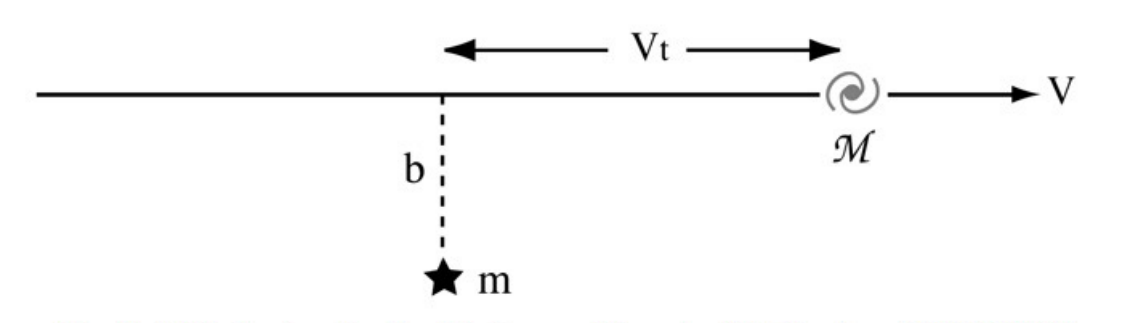
Dwarf Galaxies: Dynamical Friction



Imagine a satellite galaxy of mass M passing by a star of mass m. The perpendicular force on the satellite is

$$\vec{F}_\perp = \frac{GmMb}{(b^2 + V^2t^2)^{3/2}} = M\frac{d\vec{V}_\perp}{dt}$$

which we can integrate over time to get a change in perpendicular velocity:

$$\Delta V_\perp = \frac{1}{M}\int_{-\infty}^{\infty} \vec{F}_\perp(t)dt = \frac{2Gm}{bV}$$

We must conserve momentum, so the star must also pick up a (much bigger!) $\Delta V_\perp$ from the satellite. This means the total change in perpendicular kinetic energy is:

$$\Delta KE_\perp = \underbrace{\frac{1}{2}M\left(\frac{2Gm}{bV}\right)^2}_{\text{change in galaxy's KE}} + \underbrace{\frac{1}{2}m\left(\frac{2GM}{bV}\right)^2}_{\text{change in star's KE}} = \frac{2G^2mM(m+M)}{b^2V^2}$$

This must come from the parallel kinetic energy of the system. If we balance kinetic energy before and after the encounter:

$$\underbrace{\frac{1}{2}MV^2}_{\text{original KE}} = \underbrace{\Delta KE_\perp}_{\text{total}\perp\text{ KE}} + \underbrace{\frac{1}{2}M(V+\Delta V_\parallel)^2}_{\text{galaxy's new }\parallel\text{ KE}} + \underbrace{\frac{1}{2}m\left(\frac{M}{m}\Delta V_\parallel\right)^2}_{\text{star's new }\parallel\text{ KE}}$$

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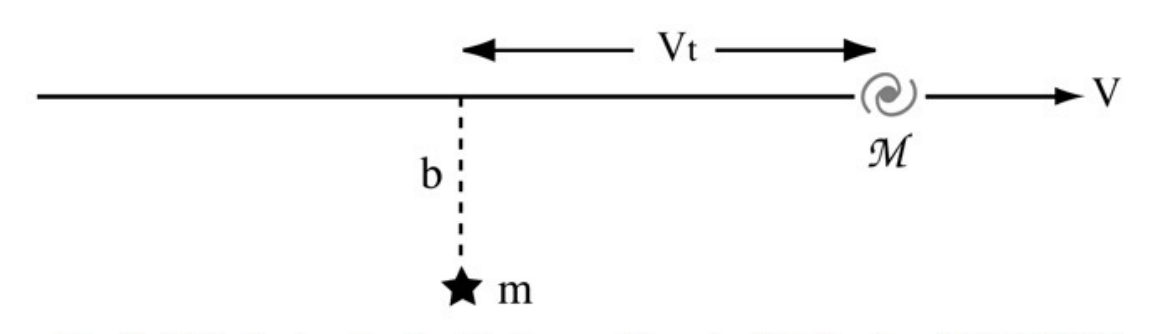

so we had: $\frac{1}{2}MV^2 = \Delta KE_\perp + \frac{1}{2}M(V+\Delta V_\parallel)^2 + \frac{1}{2}m\left(\frac{M}{m}\Delta V_\parallel\right)^2$

expand/collect terms and divide by V^2 to get

$$\frac{\Delta KE_\perp}{V^2} + \frac{M\Delta V_\parallel}{V} + \frac{1}{2}\left(\frac{\Delta V_\parallel}{V}\right)^2 + \frac{1}{2}\frac{M^2}{m}\left(\frac{\Delta V_\parallel}{V}\right)^2 = 0$$

if $\Delta V_\parallel < V$, drop terms in $(\Delta V_\parallel/V)^2$ to find that each star m slows the dwarf galaxy by an amount

$$-\Delta V_\parallel \approx \frac{\Delta KE_\perp}{MV} = \frac{2G^2m\,(m+M)}{b^2V^3}$$

if the density of stars of mass m is n stars per cubic parsec, we can integrate over all these encounters to get

$$-\frac{dV}{dt} = \int_{b_{min}}^{b_{max}} nV\,\frac{2G^2m\,(m+M)}{b^2V^3}\,2\pi b db = \frac{4\pi G^2(m+M)}{V^2}nm\ln\Lambda$$

rate of encounters
$\Delta V_\parallel$ per encounter
probability of encounter

"Coulomb logarithm":

$$\Lambda = \left(\frac{b_{max}}{b_{min}}\right)$$

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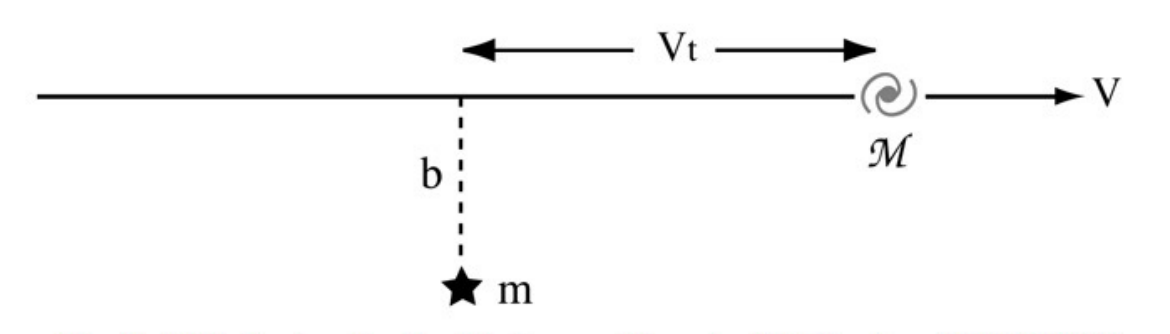

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if $M \gg m$ and we re-write the density of stars as $\rho = nm$, we get:

$$-\frac{dV}{dt} = \frac{4\pi G^2 M \rho}{V^2} \ln \Lambda$$

The Coloumb logarithm: $\ln \Lambda = \ln\left(\frac{b_{max}}{b_{min}}\right)$

b_{max} is essentially the size of the large galaxy being orbited (i.e., the size of the system of stars of mass m).

When we did it for star-star scattering, b_{min} was the close scattering radius for a star of mass m: $b_{min} \approx \frac{2Gm}{V^2} \approx 1\ AU$

But here it is larger, since we are dealing with a galaxy of mass M: $b_{min} \approx \frac{2GM}{V^2} \approx$ kiloparsec scales for MW satellites.

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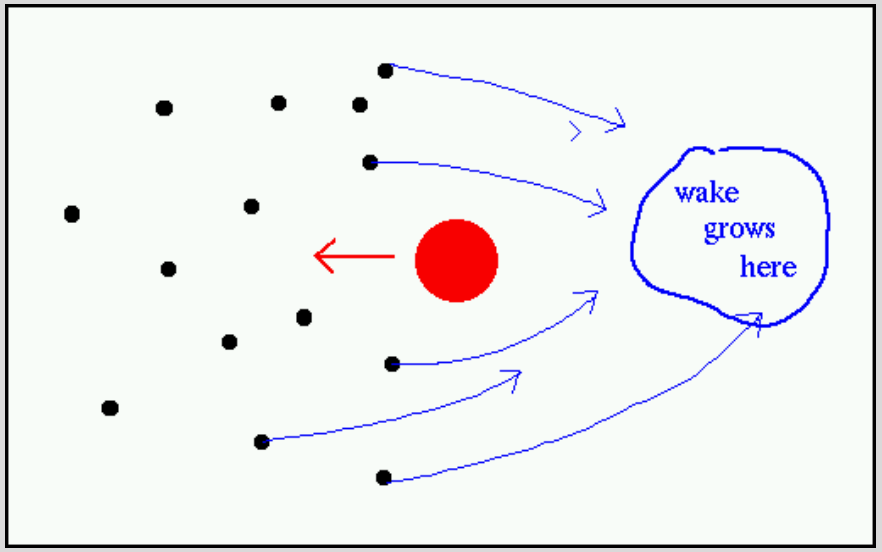
$$\frac{dV}{dt} = -\frac{4\pi G^2 M\rho}{V^2}\ln\Lambda$$

Also can consider it as a wake



Things to note:

- The net result is a drag term, the satellite is slowed down.
- Massive satellites affected more than low mass
- Denser regions do more slowing
- Fast encounters are less affected
- Does it have to be stars dragging on the satellite? What else could do the dragging?

Effects:

Circularization of satellite orbits: At peri on an elongated orbit, $V_{sat} > V_{circ}$. Friction is also strongest at peri. So over time, successive "braking" of the satellite changes the orbit form elongated to circular.

Satellite inspiral: Continual friction removes energy from satellite orbit, orbit decays and satellite spirals inward.

Merging: If satellite is dense enough to survive tidal stripping, it can merge to the center of the big galaxy.