Interpreting Rotation Curves: Back to dynamics

Fundamentally, we are building observable tracers of the underlying mass density of galaxies. To understand this, we need to tie it all together with a dynamical understanding of the relationships between mass, potential, and kinematics.

A galaxy has a mass distribution given by $\rho(\boldsymbol{x})$.

The gravitational potential is connected to density via Poisson's equation:

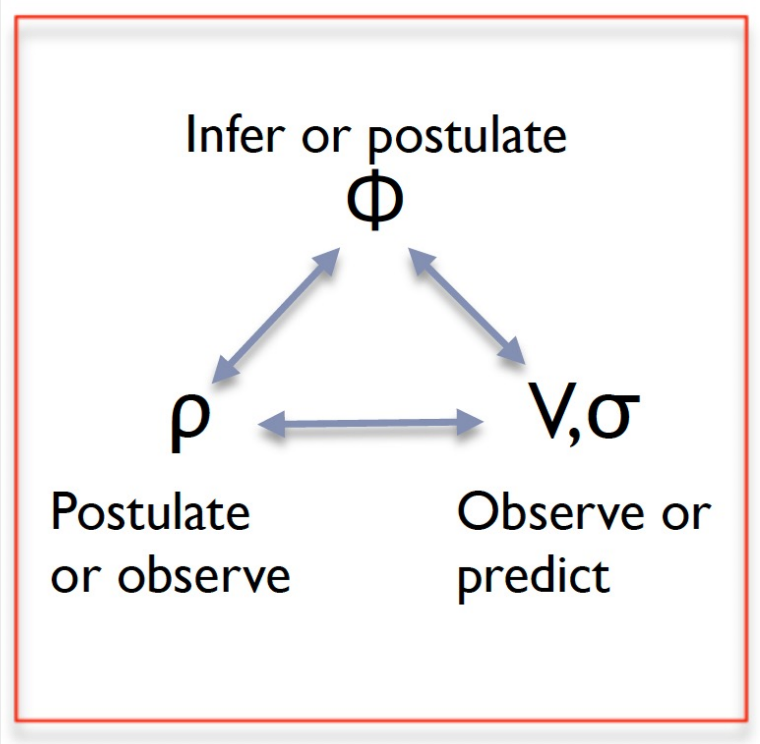
$$\nabla^2\phi = 4\pi G\rho$$

Acceleration (i..e., motion) is derived from potential via

$$\mathbf{F} = \mathrm{m}\mathbf{a} = m\boldsymbol{\nabla\phi}$$

For **spherical** mass distributions, we can solve Poisson's equation as

$$\Phi(r) = -\underbrace{\frac{G\mathcal{M}(r)}{r}}_{\text{all mass inside } r \text{ acts as a point mass}} - G\underbrace{\int_r^\infty \frac{4\pi\rho(r)r^2}{r}dr}_{\text{outside } r\text{, integrate over mass shells to get potential}}$$



courtesy Matt Bershady

Simple Example: the constant density sphere

Density

$$\rho(r) = \rho_0 \;\; \text{for}\; r < R_{max}$$

Derive mass interior to r:

$$\mathcal{M}(r) = \int_0^r \rho(r) 4\pi r^2 dr = 4\pi\rho_0 \int_0^r r^2 dr = \frac{4\pi r^3}{3}\rho_0$$

Derive circular velocity:

$$V_c^2(r) = \frac{G\mathcal{M}(r)}{r} \quad \text{or} \quad V_c(r) = \sqrt{\frac{4\pi G\rho_0}{3}r}$$

Derive potential:

$$\Phi(r) = -\frac{G\mathcal{M}(r)}{r} - G\int_r^{R_{max}} \frac{4\pi\rho(r)r^2}{r}dr$$
$$= -\frac{4\pi G\rho_0}{3}r^2 - 4\pi G\rho_0 \int_r^{R_{max}} r dr$$
$$= -4\pi G\rho_0 \left[\frac{R_{max}^2}{2} - \frac{r^2}{6}\right]$$

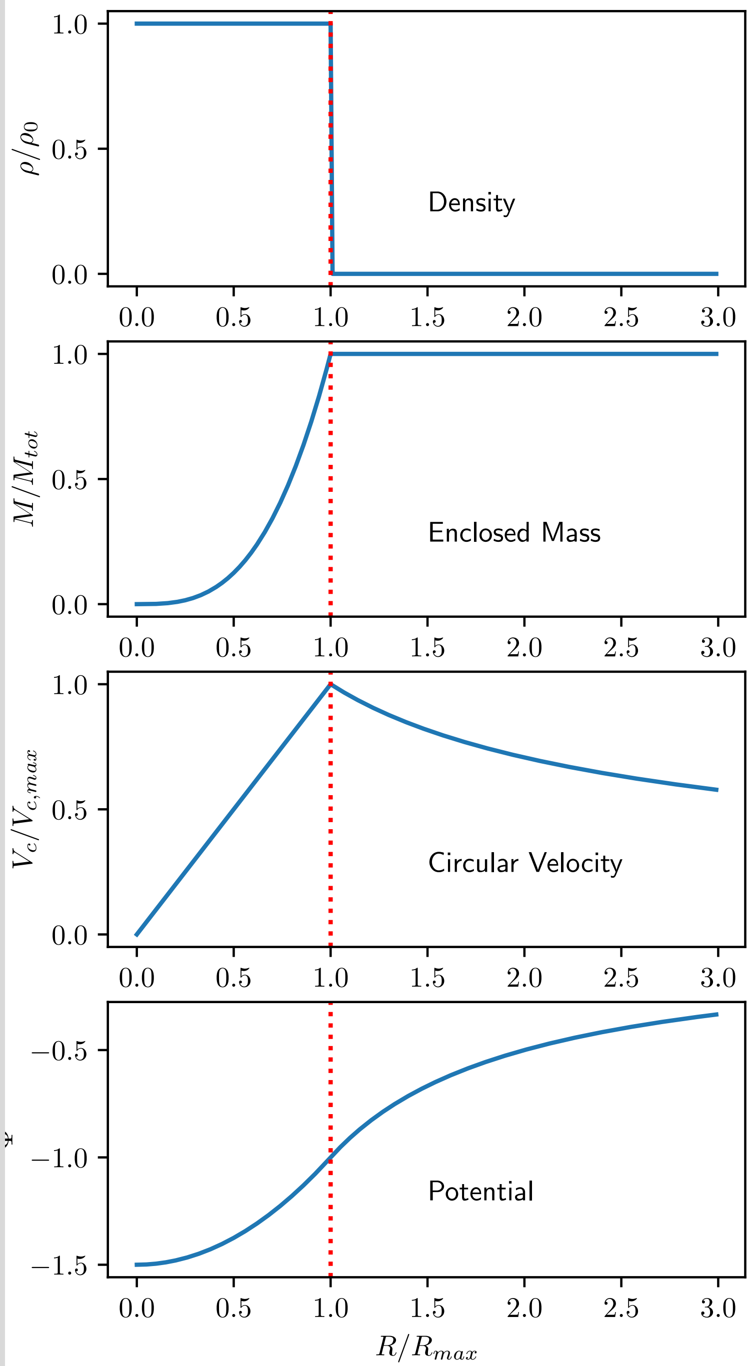
Outside R_{max} :

$$\rho(r) = 0$$

$$\mathcal{M}(r) = \frac{4\pi R_{max}^3}{3}\rho_0$$
$$= \mathcal{M}_{tot}$$

$$V_c(r) = \sqrt{\frac{G\mathcal{M}_{tot}}{r}}$$

$$\Phi(r) = -\frac{G\mathcal{M}_{tot}}{r}$$



Mass Modeling Rotation Curves

- Need to measure velocity (V_c) and know distance (to turn angular radial scale into physical scale).

- Need to have good surface brightness profile of the disk and bulge: $\mu_d(R)$ and $\mu_b(R)$

- Need to convert light to mass via a ***stellar*** mass-to-light ratio $(\mathcal{M}/L)_*$ This depends on the stellar populations and will be different for the disk and bulge, and almost certainly also a function of radius.

- Need to measure gas content: neutral hydrogen is easy, need to correct for associated helium and molecular gas.

- Need to adopt a mass model for the dark matter halo, using theoretical profiles:

Pseudo-Isothermal Halo: $\rho(r) = \frac{\rho_0}{1+\left(\frac{r}{r_c}\right)^2}$

Navarro-Frenk-White: $\rho(r) = \frac{\rho_{crit}\delta_c}{\left(\frac{r}{r_s}\right)\left(1+\frac{r}{r_s}\right)^2}$

Einasto: $\rho(r) = \rho_c exp\left[-2n\left(\left(\frac{r}{r_c}\right)^{1/n} - 1\right)\right]$

Observe, correct for inclination, adopt distance

Observe μ_b, adopt $(M/L)_{*,b}$

$$V_c^2 = V_b^2 + V_d^2 + V_{gas}^2 + V_h^2$$

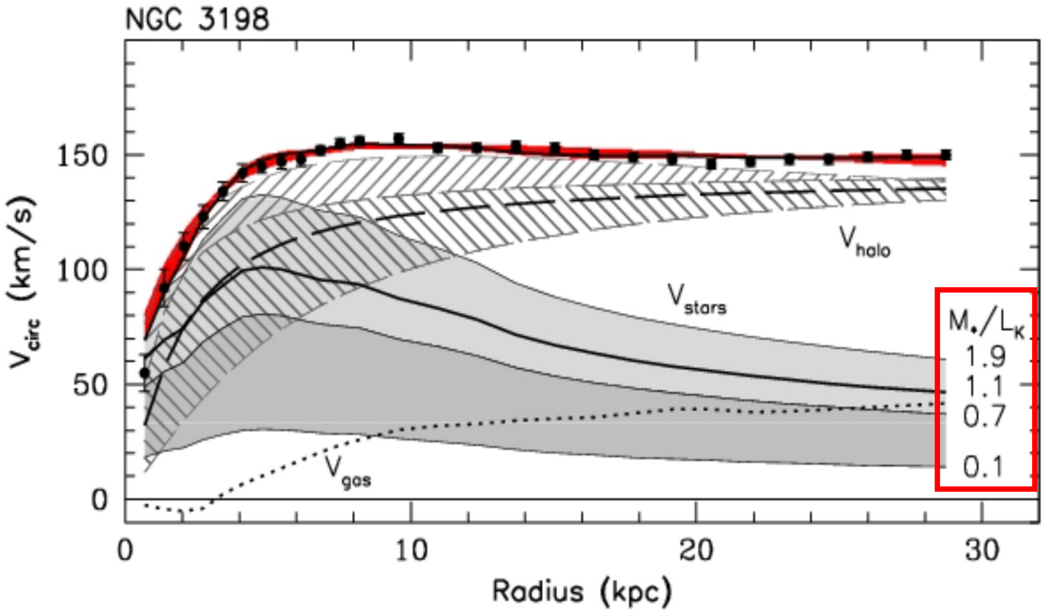
Observe μ_d, adopt $(M/L)_{*,d}$

Observe HI(+CO) make corrections

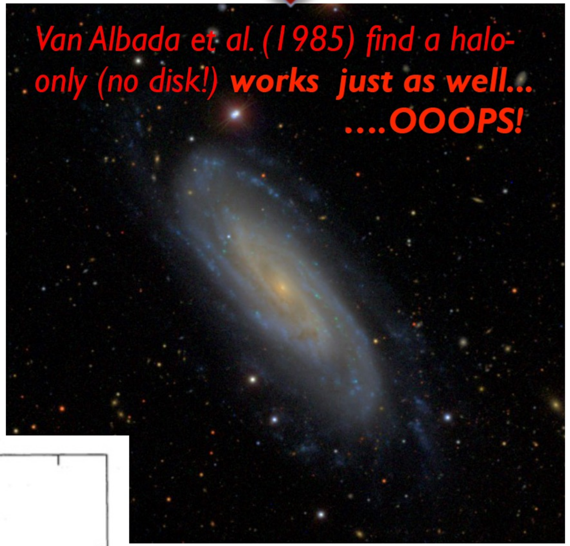
Adopt profile and fit parameters

The Disk-Halo Degeneracy: Best case

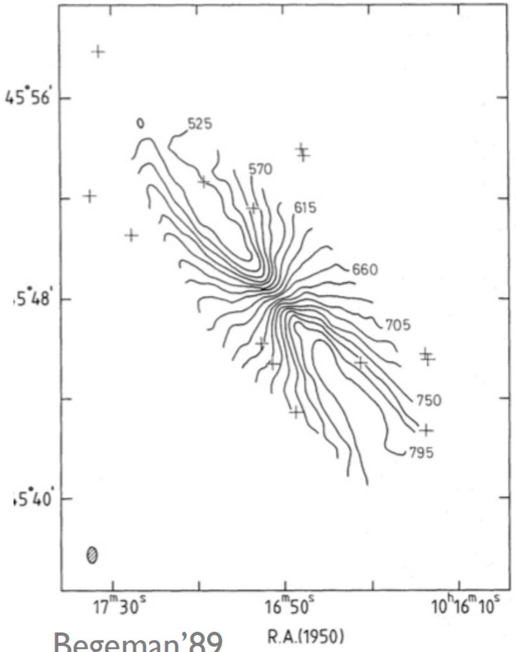
- Rotation curve decomposition constraints:



← ***Degenerate solutions…***



SDSS gri



Begeman'89

…it doesn't get better than this

←HI velocity field

courtesy
Matt Bershady
(UWisc)

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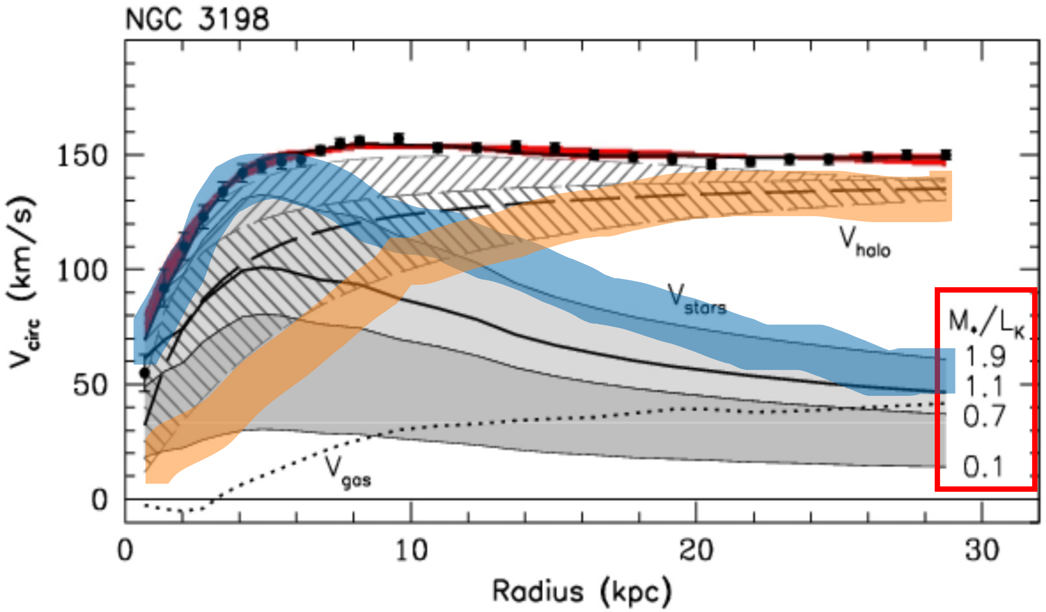

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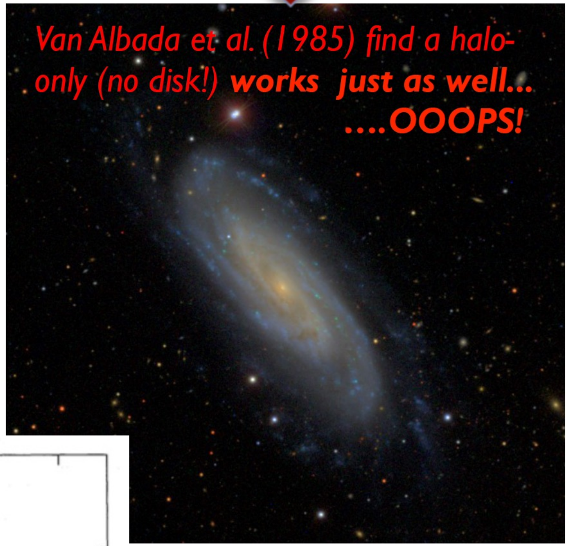

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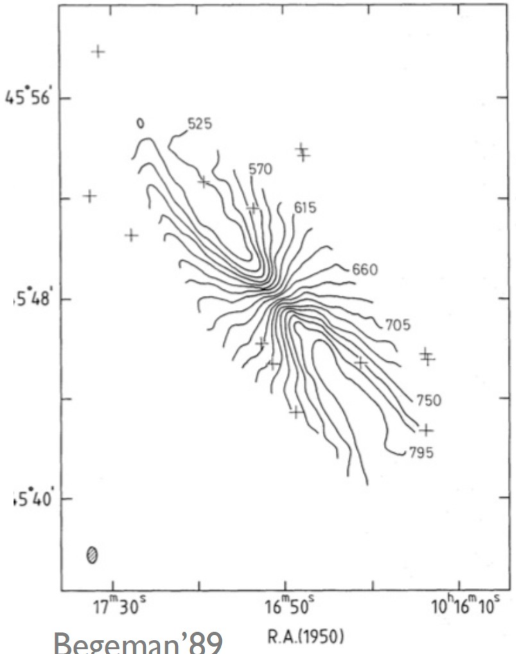

Begeman'89

…it doesn't get better than this

←HI velocity field

If $(M/L)_*$ is high, inner rotation is all due to disk mass. Outer parts still need halo component.

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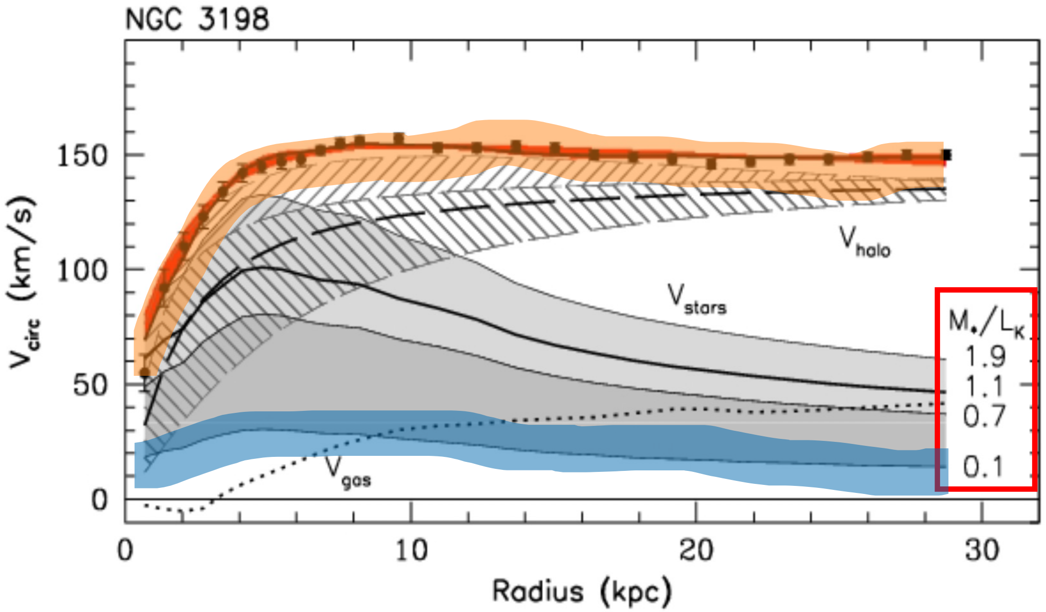
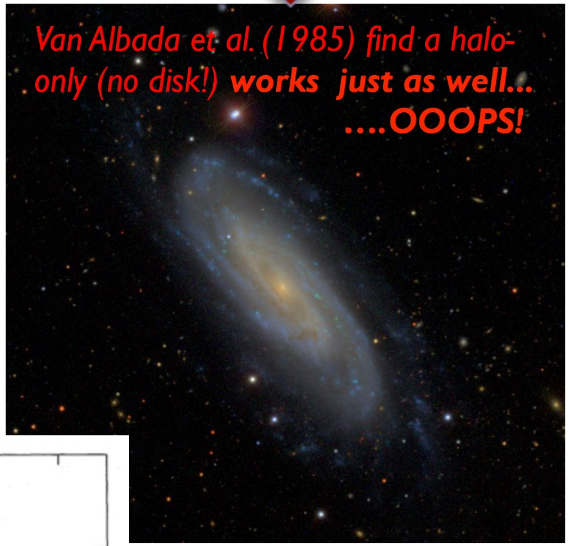
Matt Bershady
(UWisc)

The Disk-Halo Degeneracy: Best case

- Rotation curve decomposition constraints:



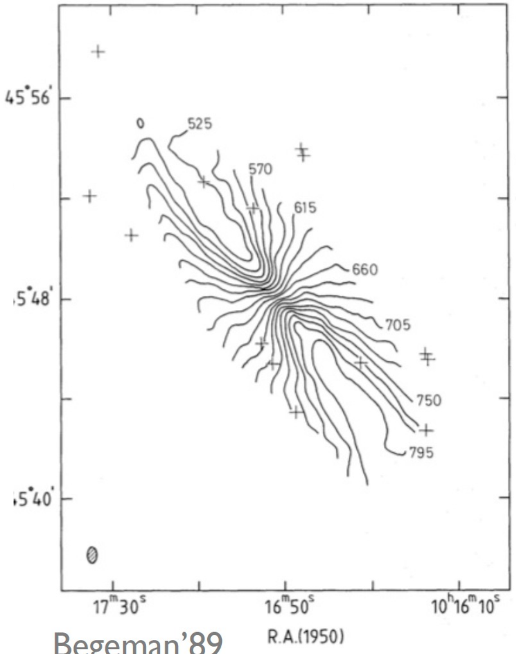
← ***Degenerate solutions…***



SDSS gri

But if $(M/L)_*$ is low, the disk has less mass and the halo dominates throughout.

Disk/Halo Degeneracy: *we need to know $(M/L)_*$ very accurately to break this degeneracy!*



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Spiral structure in disks

Types of spirals:

Grand design: 2 well-defined, symmetric spiral arms.

Flocculent: spiral arm “fragments”, not continuous

Multiple arms: 3, 4, etc

Barred spirals

M81 (Adam Block)

Spiral structure in disks

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Grand design: 2 well-defined, symmetric spiral arms.

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Spiral structure in disks

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Multiple arms: 3, 4, etc

Barred spirals


NGC 5054 (Michael Sidonio)

Spiral structure in disks

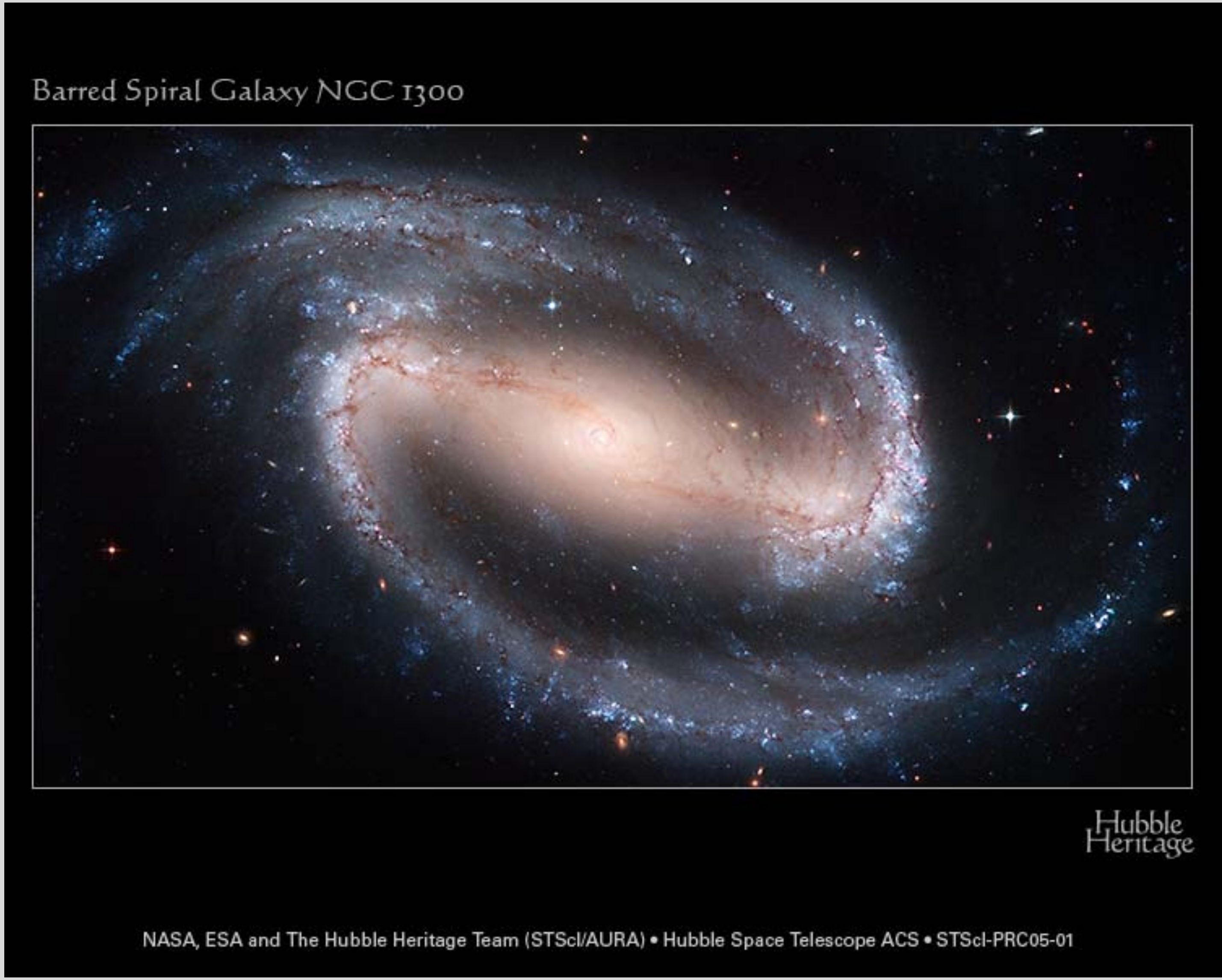
Types of spirals:

Grand design: 2 well-defined, symmetric spiral arms.

Flocculent: spiral arm “fragments”, not continuous

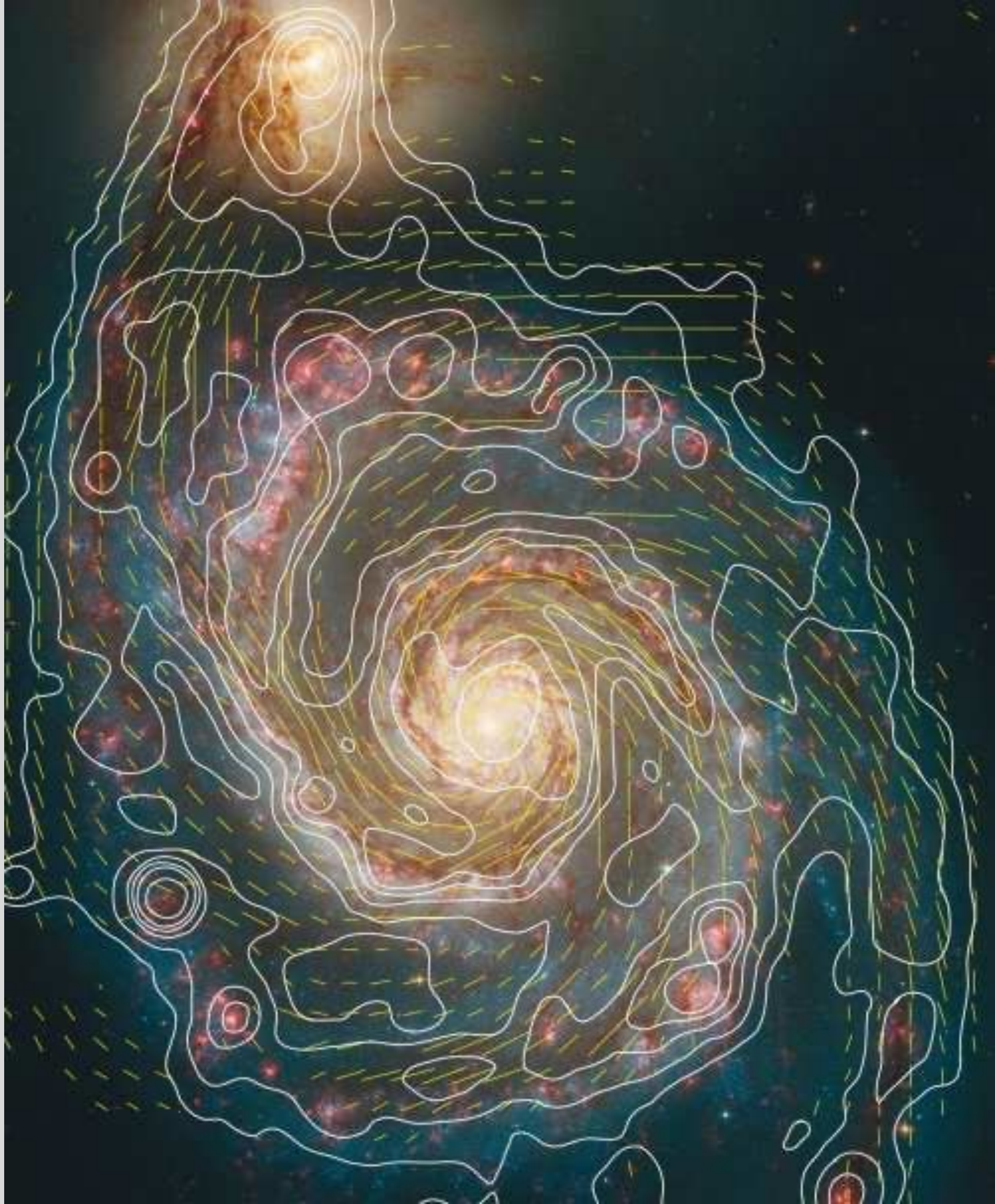
Multiple arms: 3, 4, etc

Barred spirals: arms coming off a central bar



Properties of Spirals

Very prominent at blue wavelengths, in Hα emission, and in radio continuum: star formation tracers.



Color image: optical/Hα
Countours: radio continuum

Properties of Spirals

Very prominent at blue wavelengths, in Hα emission, and in radio continuum: star formation tracers.

Strong dust lanes (often inside the arms): shows where gas enters the spiral arm.

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Strong dust lanes (often inside the arms): shows where gas enters the spiral arm.

Velocity perturbations of ~ 20-30 km/s along spiral arms: arms are a significant enhancement of mass.

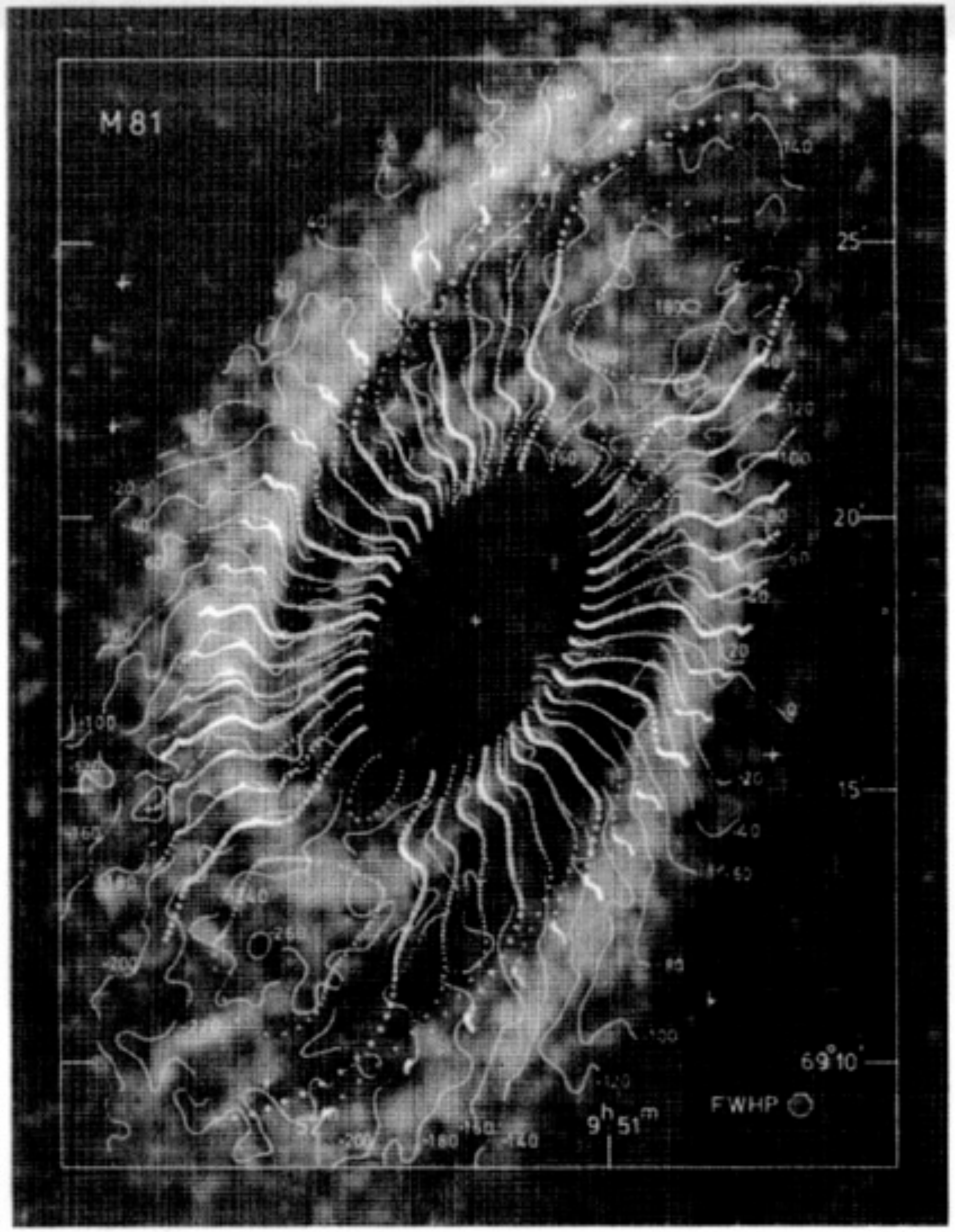


Figure 6-23. Constant-velocity contours of HI in the spiral galaxy M81. Solid lines represent observations made at the Westerbork Synthesis Radio Telescope, while chains of symbols represent predictions of a model based on the Lin-Shu hypothesis (Visser 1980). The contours are superimposed on an artificial photograph in which brightness is proportional to HI column density. The shaded circle at lower right represents the spatial resolution of the observations.

Properties of Spirals

Very prominent at blue wavelengths, in Hα emission, and in radio continuum: star formation tracers.

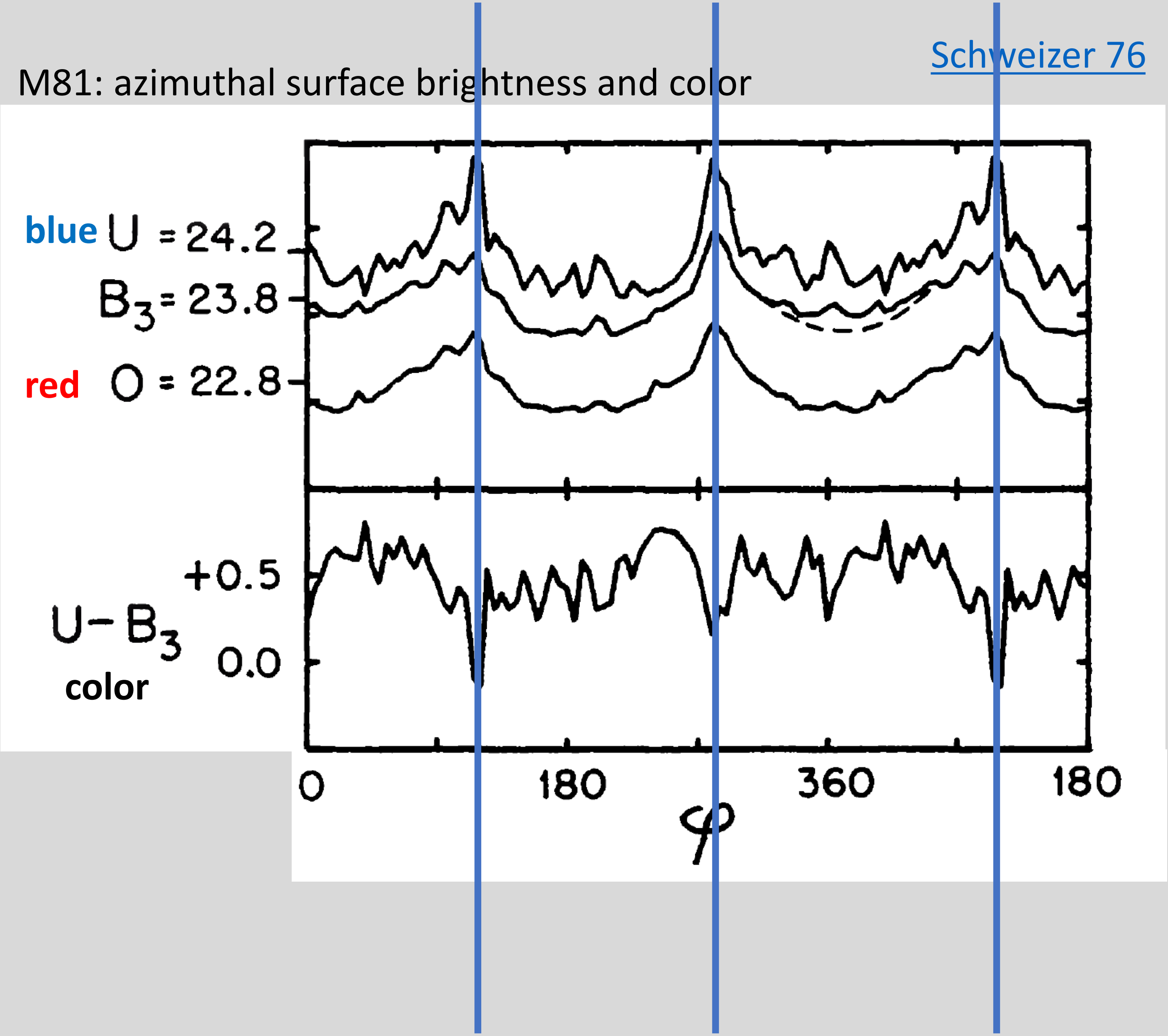
Strong dust lanes (often inside the arms): shows where gas enters the spiral arm.

Velocity perturbations of ~ 20-30 km/s along spiral arms: arms are a significant enhancement of mass.

In red light, spiral arms are smoother, broader, lower in amplitude. Red light traces older stars, showing that the entire disk participates in the spiral structure.

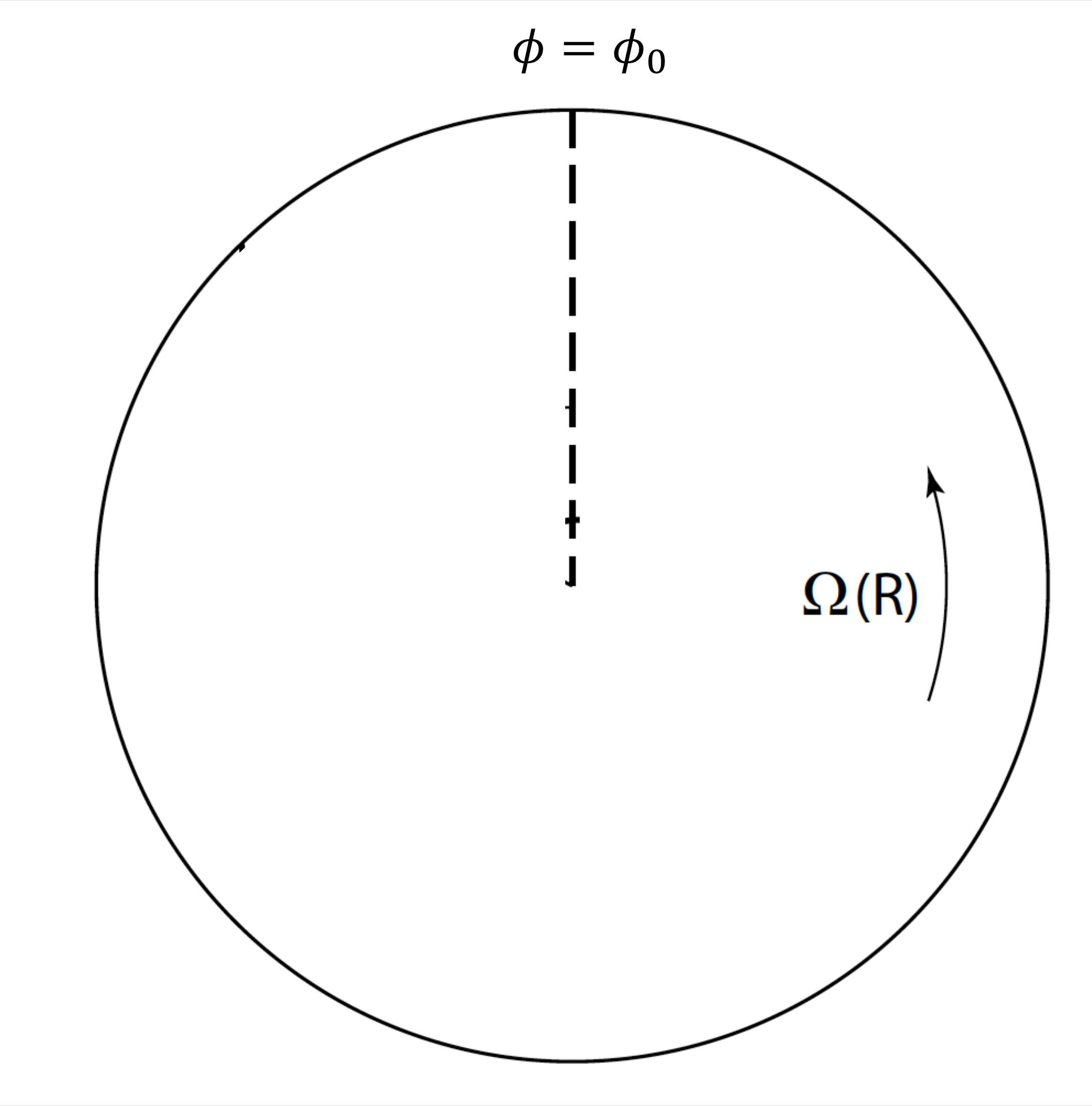
[Schweizer 76](#)

M81: azimuthal surface brightness and color



Winding Problem

Imagine painting a radial stripe on a rotating galaxy at some angle ϕ_0.



Winding Problem

Imagine painting a radial stripe on a rotating galaxy at some angle ϕ_0. After some time t, that stripe will “wind up” and follow the equation

$$\phi(R,t) = \phi_0 + \Omega(R)t$$

where $\Omega(R) = V(R)/R$ is the angular rotation frequency.

The spiral has a pitch angle α defined by

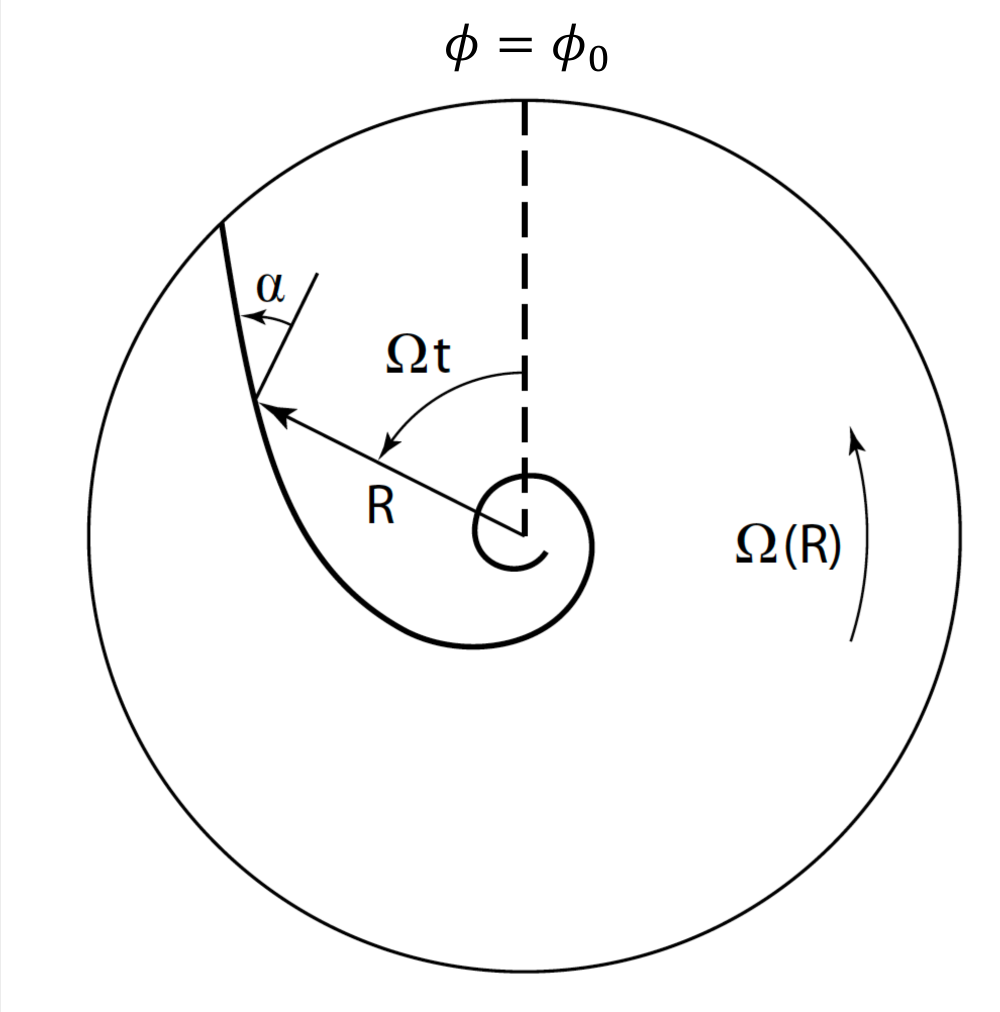
$$\cot\alpha = \left|R\frac{\partial\phi}{dR}\right| = Rt\left|\frac{\partial\Omega}{\partial R}\right|$$

remember, here ϕ refers to the arm orientation, not the gravitational potential!

If we want the stripe to stay fixed in shape (but allow it to rotate), what is the requirement for $V(R)$?

We would need a constant $\Omega(R)$, meaning $V(R) \sim R$.

Do galaxies behave this way?



Winding Problem

How fast would galaxies “wind up”?

The stripe will wrap completely at a time t where

$$2\pi = |\Omega(R + \Delta R) - \Omega(R)| \times t$$

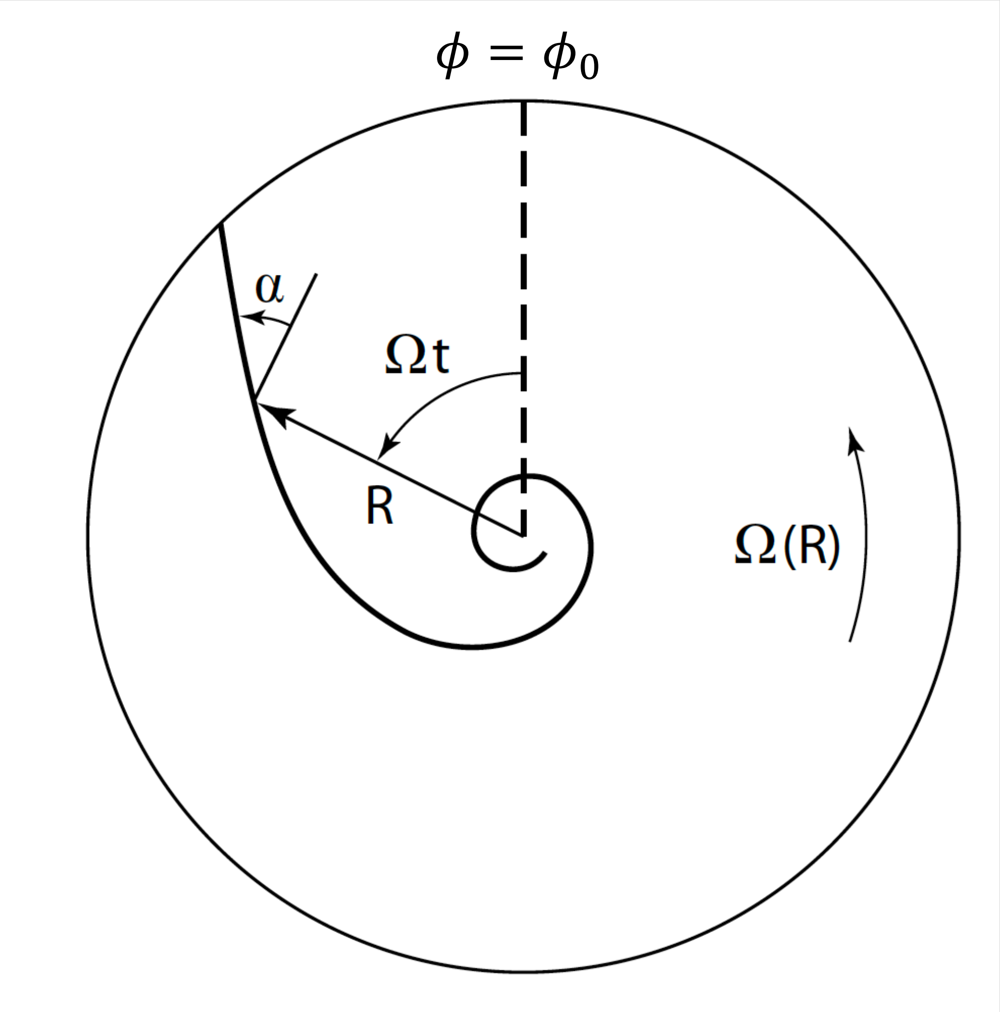
where ΔR would be the distance between wraps.

If $\Delta R \ll R,$ then $\Omega(R + \Delta R) = \Omega(R) + \frac{\partial \Omega}{\partial R} \Delta R$

$$\text{so} \quad 2\pi = \left|\frac{\partial \Omega}{\partial R} \Delta R\right| \times t$$

$$\text{or} \quad \Delta R = \frac{2\pi}{\left(\frac{\partial \Omega}{\partial R}\right) t} = \frac{2\pi R}{\cot \alpha}$$

where that last step comes from the definition of pitch angle.



Winding Problem

Put in some numbers. If

$$\Delta R = \frac{2\pi}{\left(\frac{\partial\Omega}{\partial R}\right) t} = \frac{2\pi R}{\cot\alpha}$$

Then for a Milky Way type galaxy with

- $\Omega(R)R = V_c = 220$ km/s
- $R = 10$ kpc
- $t \approx 10$ Gyr

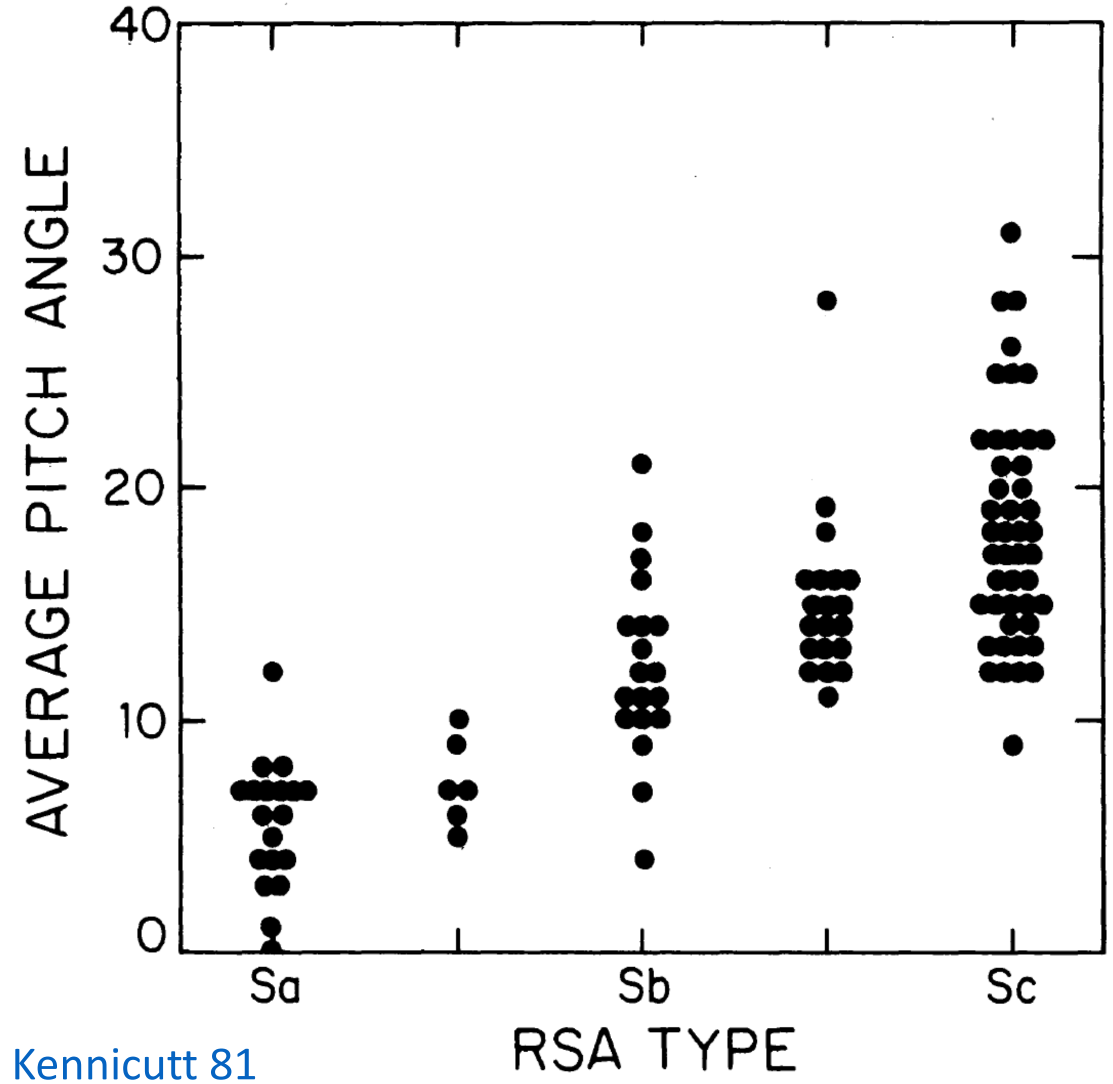
we get:

$\alpha = 0.25$ degrees
ΔR = 0.3 kpc

Hmm.... Real galaxies have much bigger pitch angles ➔

So spiral arms in galaxies are not so tightly wound!

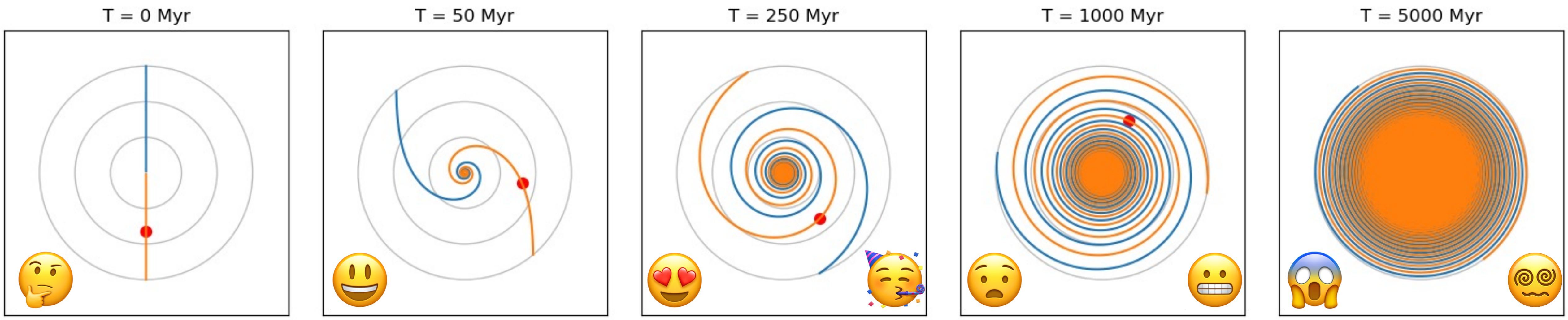
Look at observed pitch angles



Kennicutt 81

Making spiral arms

Imagine making a linear “ridge” of stars and letting it orbit around the galaxy. What happens over time?



The winding problem

Galaxies do not rotate like a solid object – since $V_c(R)$ is roughly constant with radius , the orbital time is short in the inner disk and long in the outer disk. This means any physical structure will wind up very quickly and be sheared away.

What would the rotation curve have to look like for this not to be a problem?

Orbital time is $T = \frac{2\pi R}{V_c(R)}$ so if the orbital time needs to be the same at all radius, then $V_c(R) = \frac{2\pi R}{T} \sim R$

“Solid Body Rotation”
Not what galaxies do!

Spiral Density Waves

Spirals cannot be physical structures orbiting coherently for long timescales. Instead, they are density waves moving through the disk. What is a density wave?

A traffic jam is an example of a density wave. Cars move in and out of the jam at a different speed than the jam itself moves.

Density Waves

Ω_p is called the **pattern speed.**

Spiral arms can't be physical arms – they would wind up too quickly.

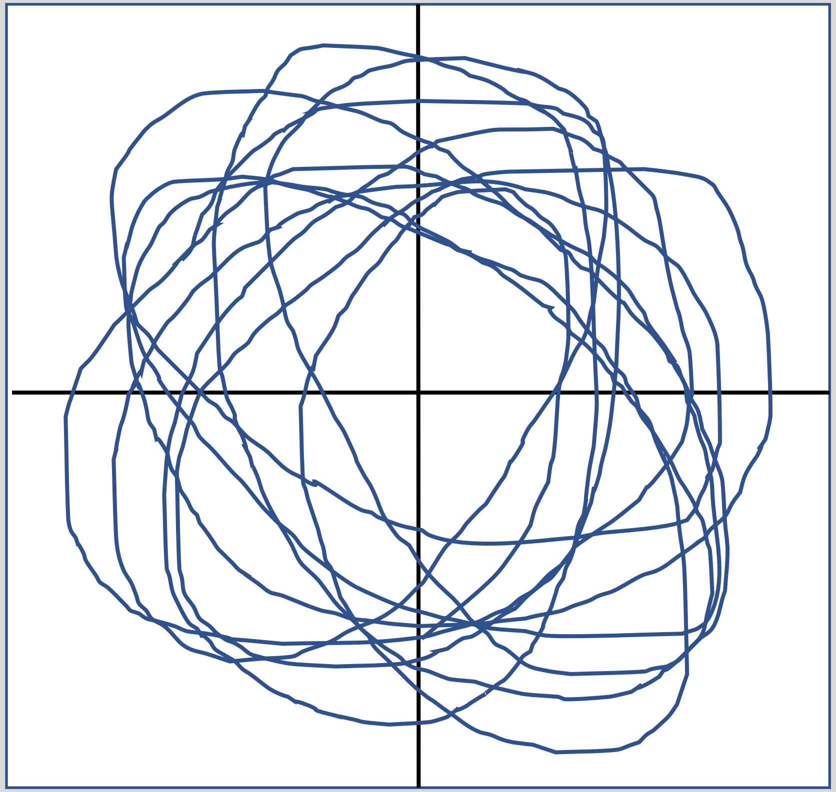
Instead consider a "density wave" – a pattern that moves through the disk at a frequency $\Omega_p < \Omega_{orb}$. Individual stars move in and out of the pattern as they orbit the galaxy, but their orbits are coordinated in such a way to sustain the pattern.

Q: How can orbits be coordinated to make a pattern? Need to think in terms of how orbits look in a rotating frame of reference.

Remember the important frequencies of orbits:

- Ω: orbital frequency (V/R)
- κ: epicyclic frequency

Viewed in a **non-rotating frame**, orbits in galactic potentials are open rosettes, since Ω/κ is generally non-integer.



Non-rotating frame

Density Waves

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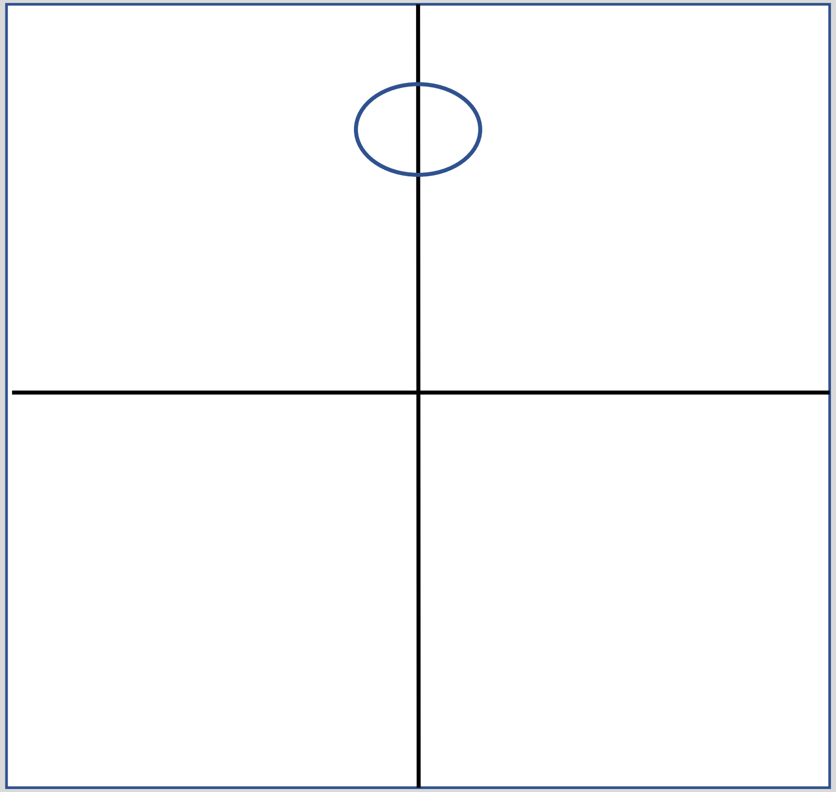
Remember the important frequencies of orbits:

- Ω: orbital frequency (V/R)
- κ: epicyclic frequency

Viewed in frame **rotating at $\boldsymbol{\Omega_p = \Omega}$**, we saw the orbit showed the epicyclic motion.

(Here the rotating frame shows you how stars move relative to average circular motion as the disk rotates.)

Frame rotates at $\Omega_p = \Omega$



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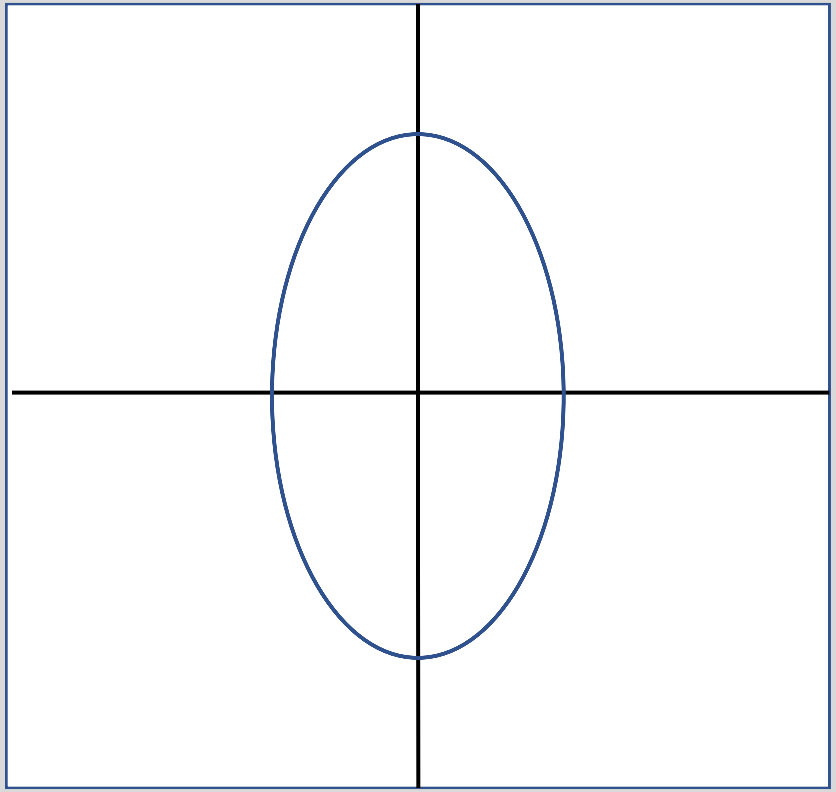
Q: How can orbits be coordinated to make a pattern? Need to think in terms of how orbits look in a rotating frame of reference.

Remember the important frequencies of orbits:

- Ω: orbital frequency (V/R)
- κ: epicyclic frequency

Viewed in frame **rotating at $\boldsymbol{\Omega_p = \Omega - \kappa/2}$**, the orbit appears as a closed ellipse.

(Here the rotating frame shows you how stars move relative to an average motion at Ω_p.)



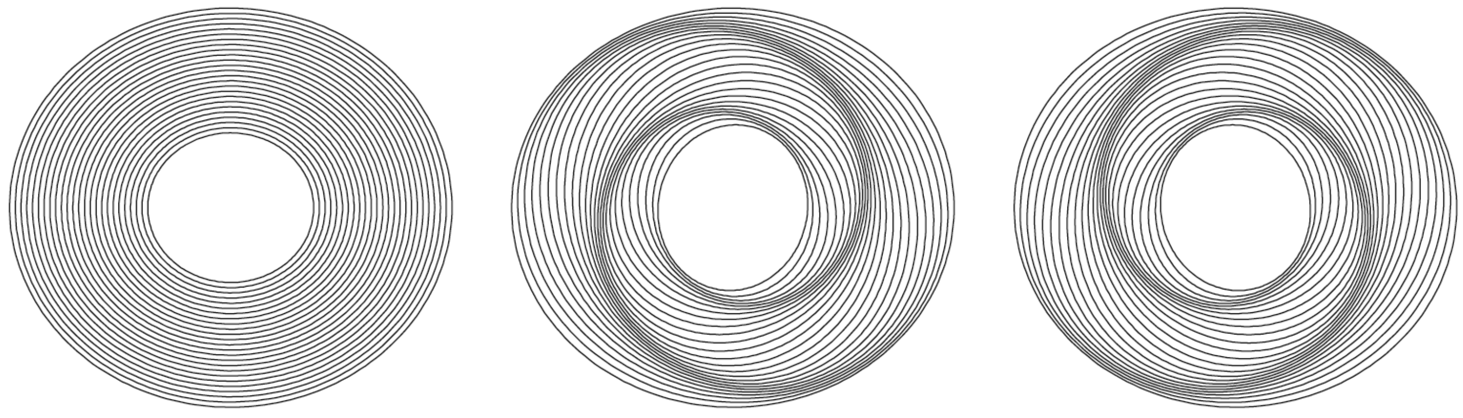
Frame rotates at $\Omega_p = \Omega - \frac{\kappa}{2}$

Density Waves

In general, viewed in frames rotating at $\Omega_p = \Omega - \frac{n}{m}\kappa$ orbits appear closed if n, m are integers.

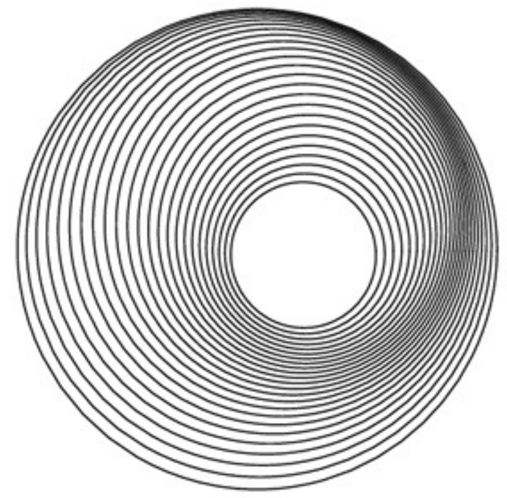
Closed orbits in a rotating frame mean that the pattern will stay in shape, but rotate slowly at a rate Ω_p, even though the stars are orbiting at a different rate of $\Omega = V_c/R$.

So we can set up nested orbits in a variety of patterns to form bars and spirals ($m = 2$):



(a) (b) (c)

Or one-armed spirals ($m = 1$):



Those spiral patterns will then rotate at a pattern speed $\Omega_p = \Omega - \frac{n}{m}\kappa$, and stars (orbiting at Ω) will move in and out of the spiral pattern.

Density Waves

Remember, Ω and κ are set by the rotation curve. So we can look at $\Omega_p = \Omega - \frac{n}{m}\kappa$ as a function of radius given a rotation curve.

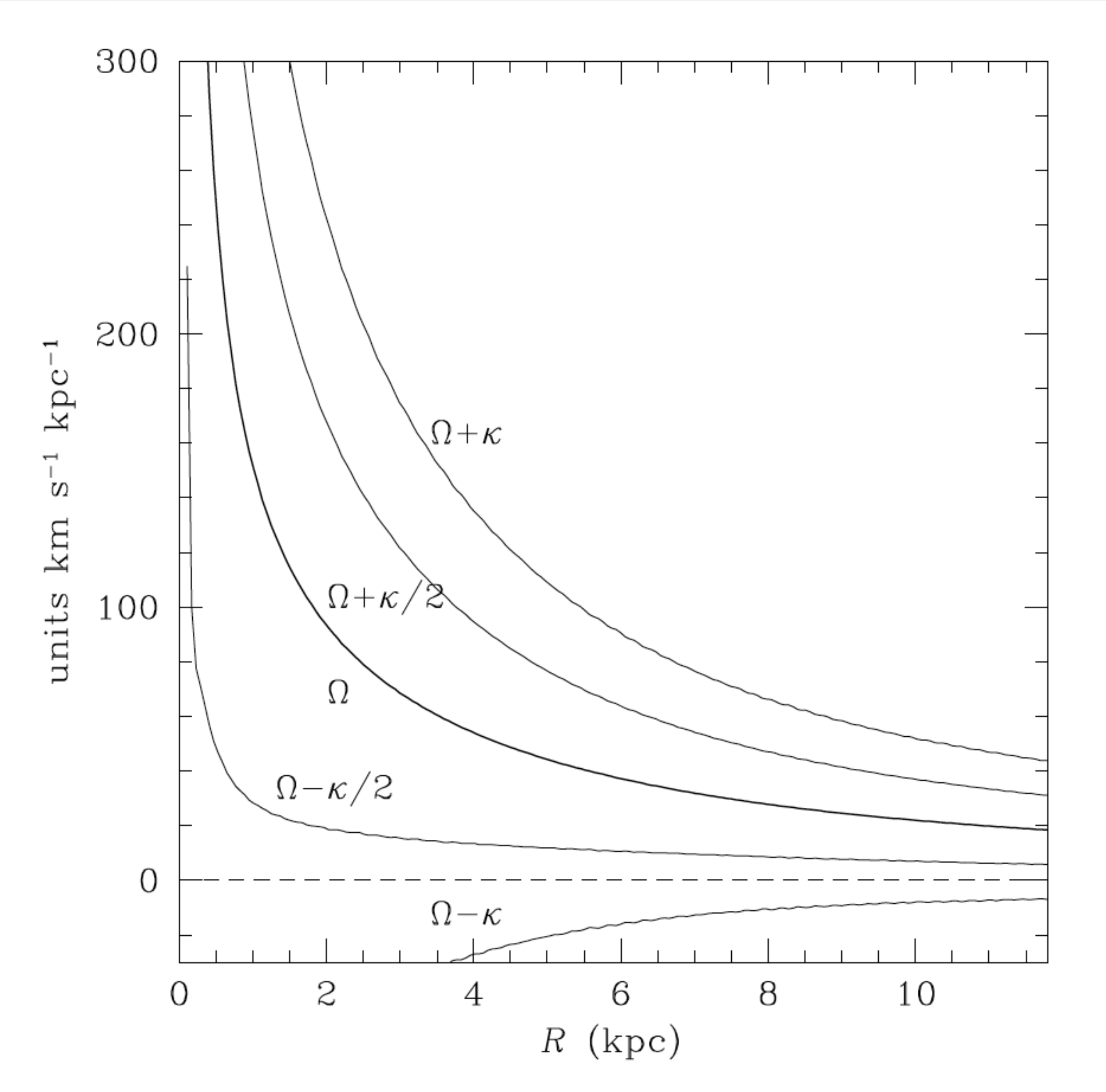
For typical rotation curves, $\Omega - \frac{\kappa}{2}$ is nearly constant over a wide range of radius.

Look at winding up via spiral arm pitch angles.

Physical arms	$\cot\alpha = Rt\left\lvert\frac{\partial\Omega}{\partial R}\right\rvert$
Density waves	$\cot\alpha = Rt\left\lvert\frac{\partial(\Omega-\kappa/2)}{\partial R}\right\rvert$

Smaller gradient means bigger pitch angles (less winding)

Since $\Omega - \frac{\kappa}{2}$ is not perfectly constant, we still have winding, but slower by a factor of ~ 5. So density waves last longer, but still must be regenerated or reinforced.



Density Waves

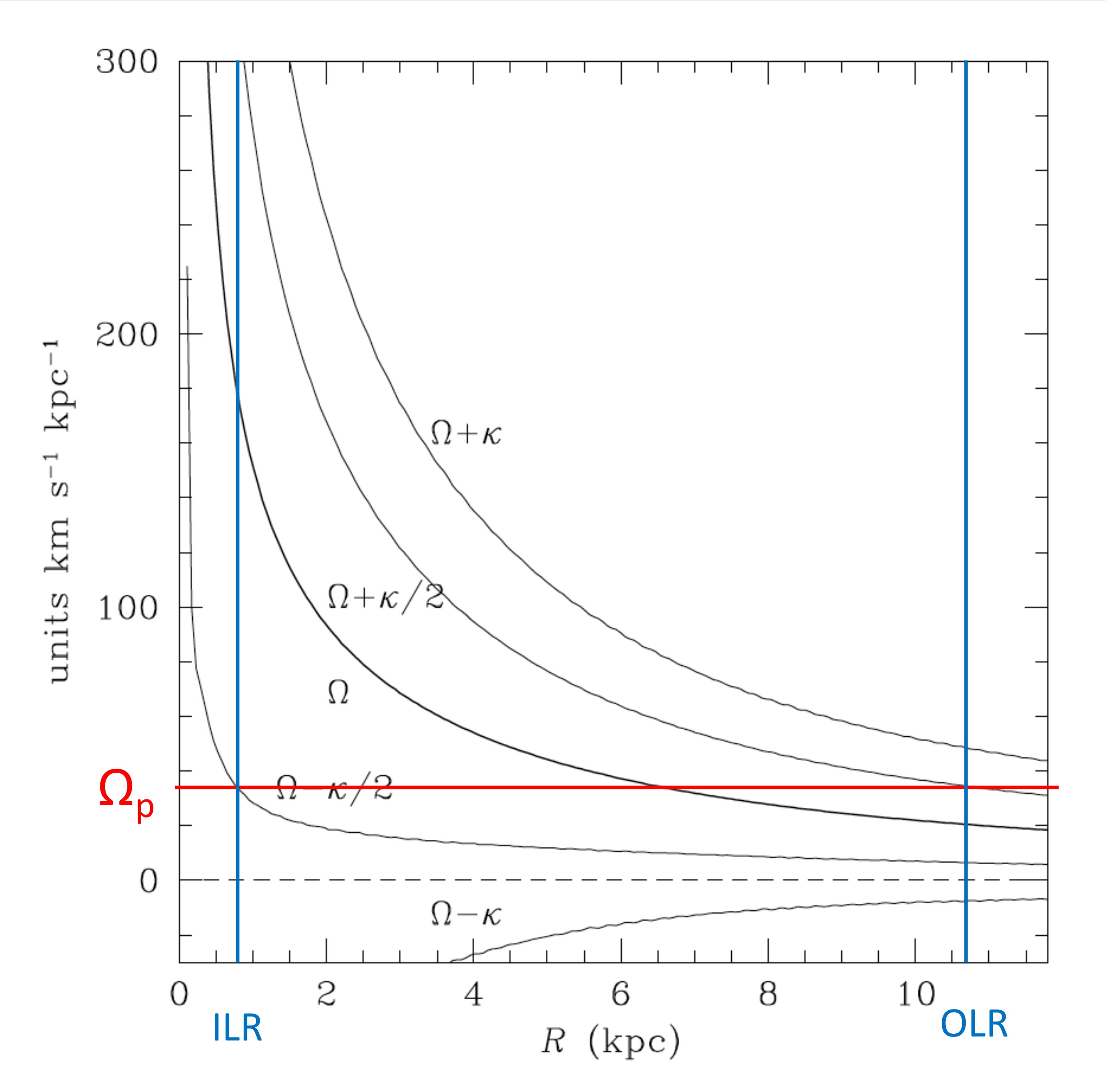
So far, we have only considered ***kinematic*** density waves (correlated motion). But as stars move through the pattern, the mass of the density wave can perturb their motion and strengthen the wave: "**self-gravity.**"

A star passes through the pattern with a frequency $m\left[\Omega_p - \Omega(R)\right]$. If that frequency is slower than the epicyclic frequency, the perturbation will strengthen the spiral pattern.

m=2 spirals reinforced only in the region where

$$\Omega - \kappa/2 < \Omega_p < \Omega + \kappa/2$$

These critical limits are known as the **Inner and Outer Lindblad Resonances**. At the LRs, a star enters the pattern each time at the same point in the epicycle. This pumps energy into the orbits of stars, destroying the wave pattern.



Spiral Density Waves: Recapping the story....

- Spiral patterns are waves moving through the disk at an angular speed Ω_p. Stars move through this wave but do not stay in it *(think cars on the freeway moving through a traffic jam....).*

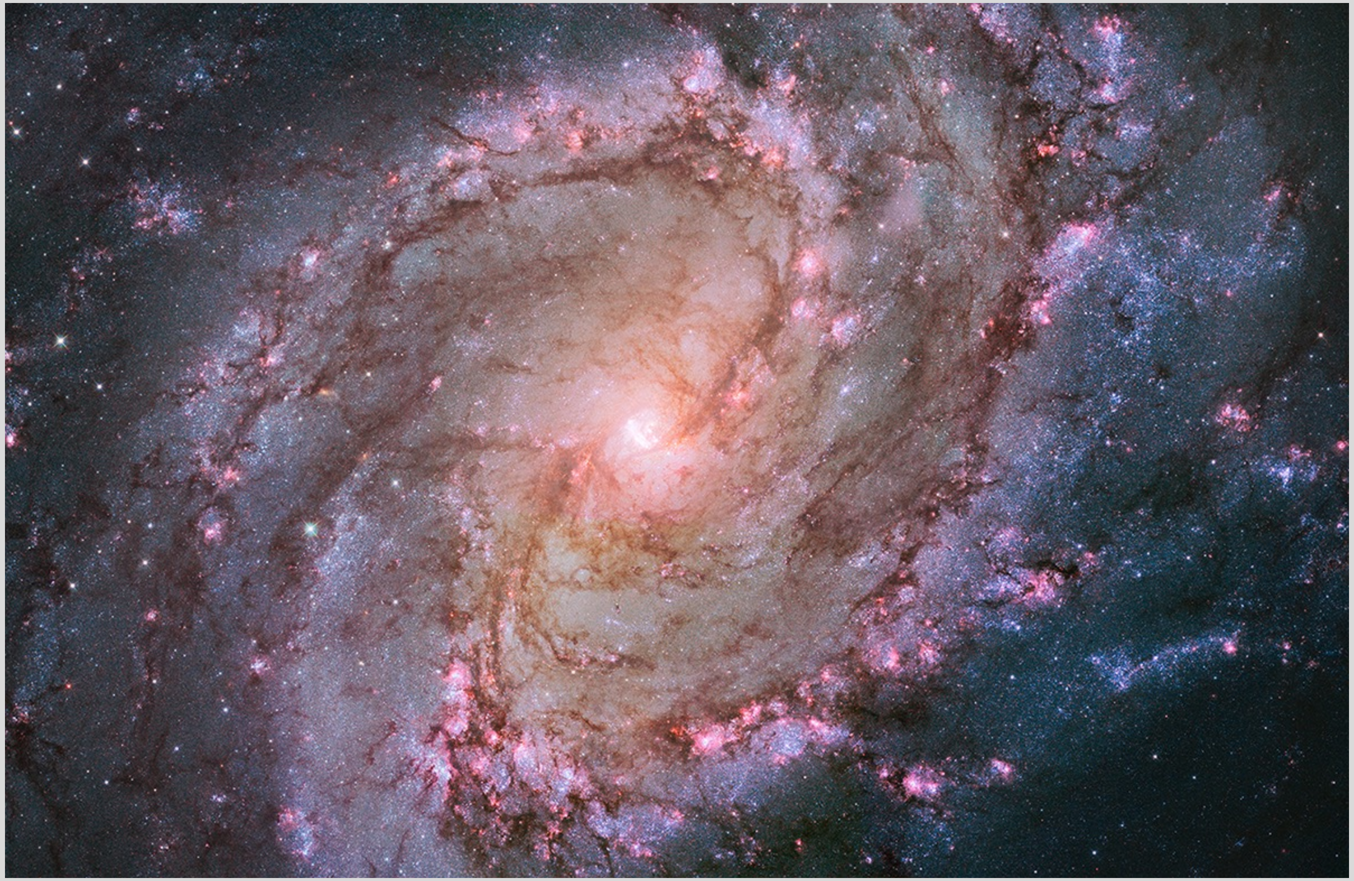
- Properties of the wave depend on the circular speed and the epicyclic frequency of the disk.

- Spiral waves can be sustained between the inner and outer Lindblad resonances.

- The gravity of the disk can amplify/sustain the spiral beyond a pure kinematic wave.

- Arms still wind up, but more slowly than expected if rotating at the rotation frequency, Ω.

- As gas moves into the spiral arms, it is shocked and driven into gravitational collapse, triggering star formation.

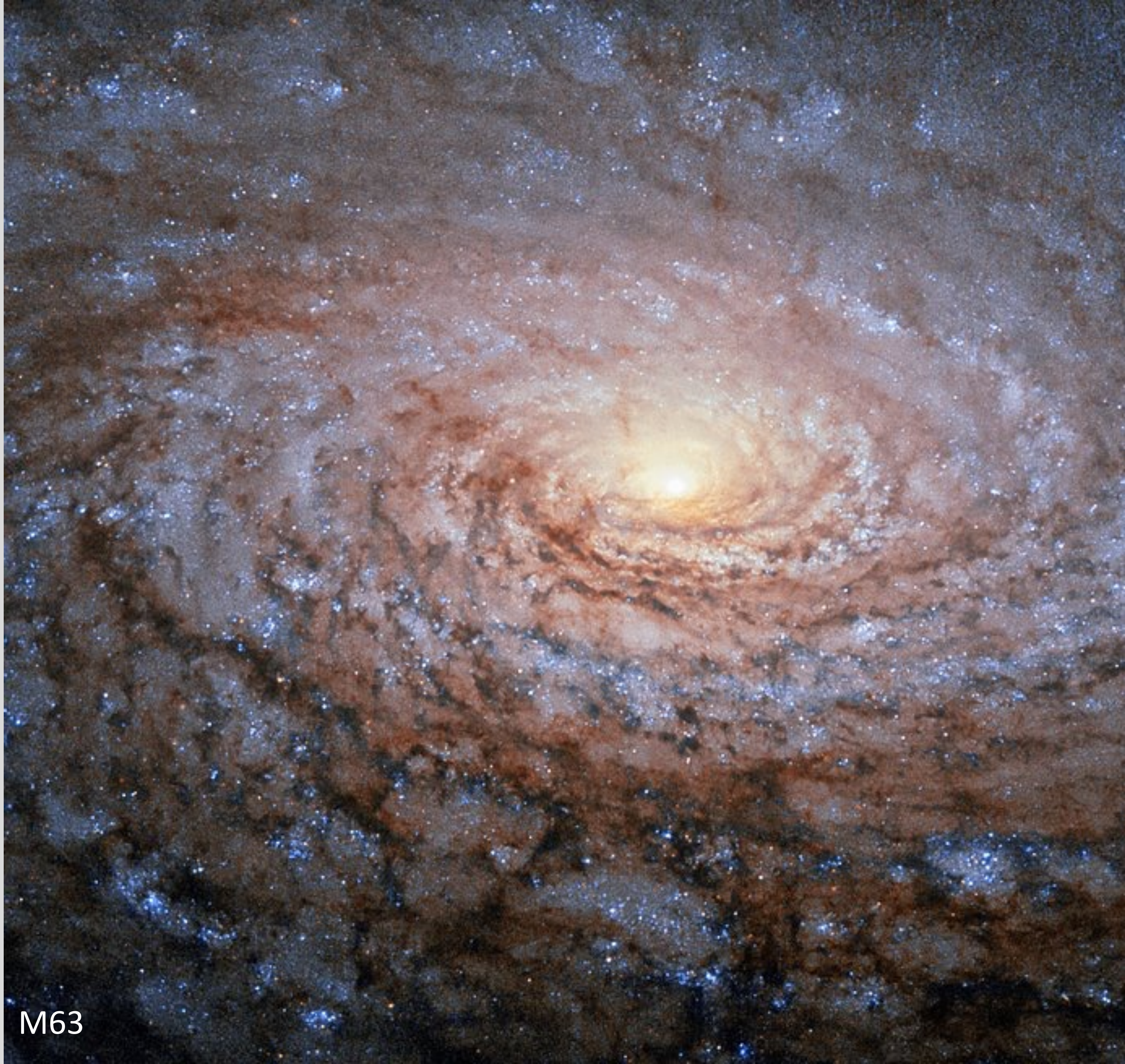


But where do they come from?

Rotational shearing: Take a patch newly formed stars, shear it out as the galaxy rotates.

Works on small scales, likely what's going on in flocculent spirals.

Not a good explanation by itself for large, organized spiral arms. But maybe self gravity can amplify the effect?



But where do they come from?

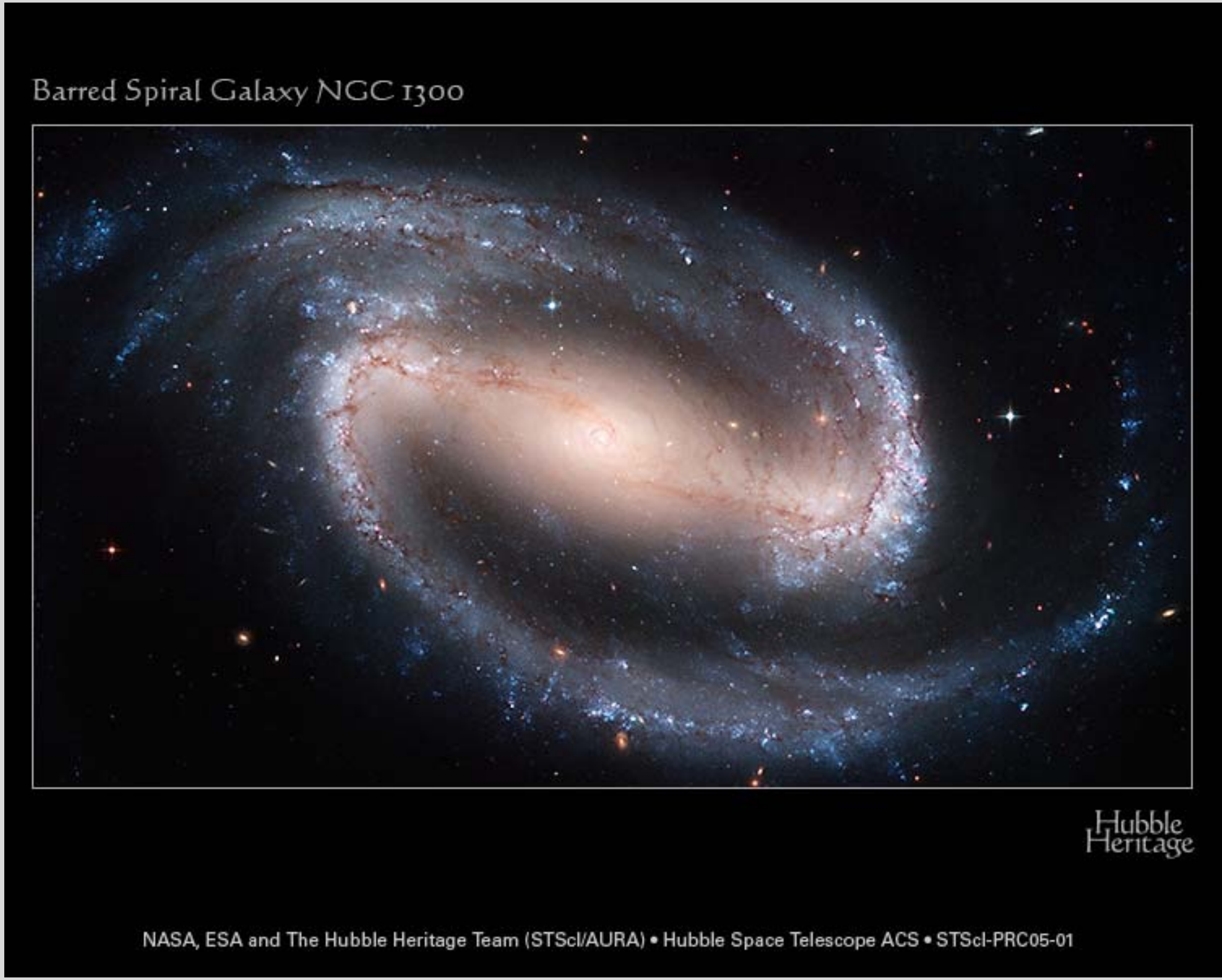
Interactions: A companion galaxy can drive a perturbation that leads to spiral structure.

But not all spirals have massive companions.

- Past encounters?
- Lower mass companions driving periodic perturbation?

But where do they come from?

Bars: The gravitational perturbation of a rotating bar may drive spiral waves in the disk.



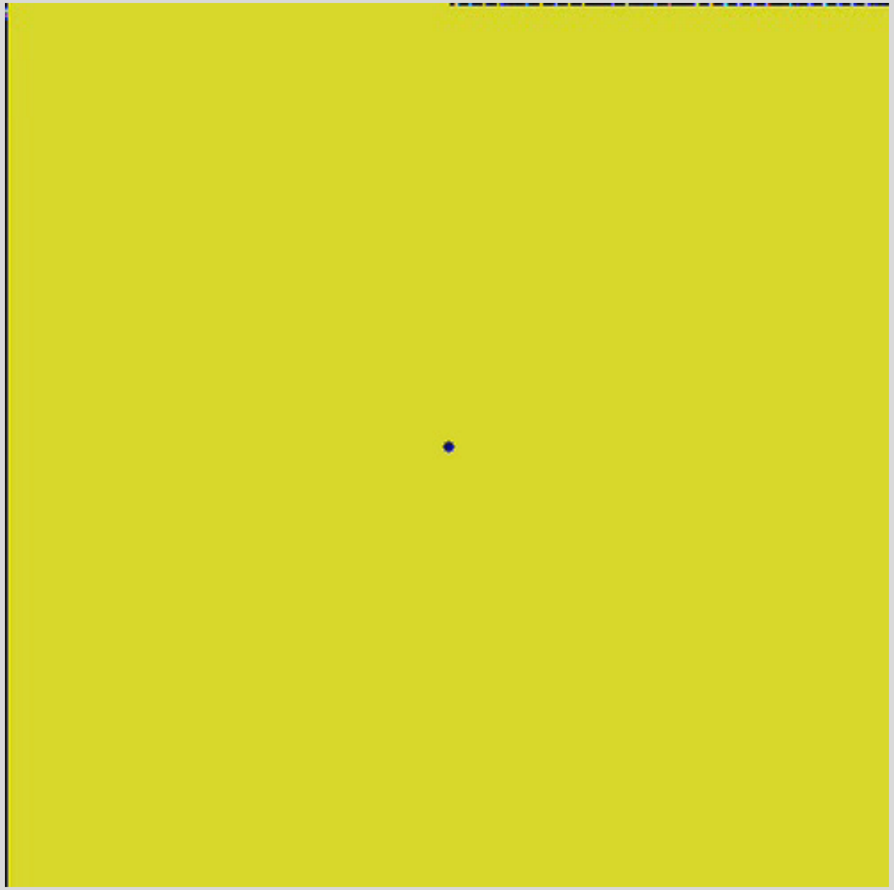
Instability and Galactic Bars

Spiral density wave scenario built on linear perturbation theory, epicyclic approximation, etc: all small amplitude deviations from axisymmetry and circular motion. What happens when the amplitude gets too strong?

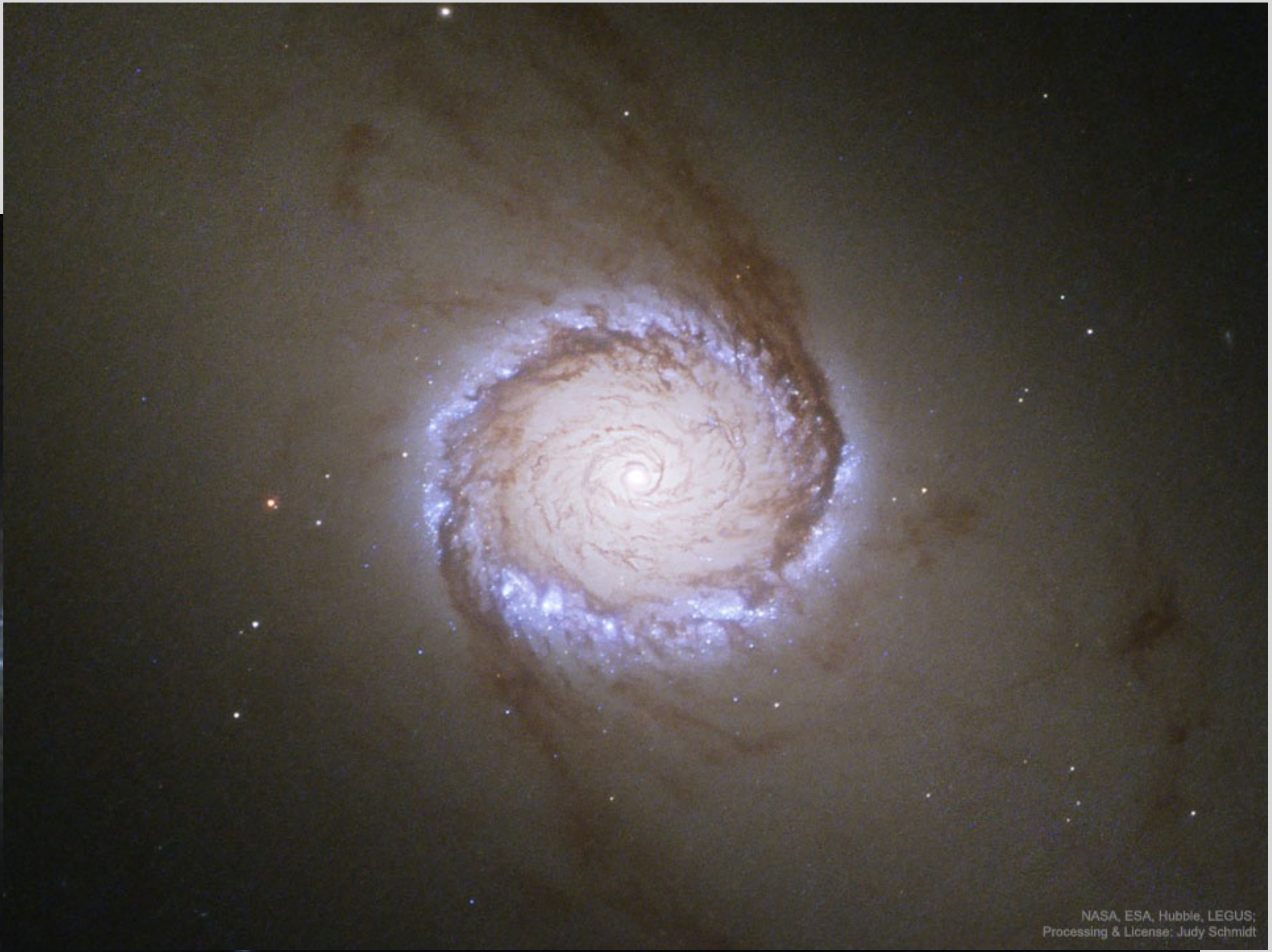
Stars no longer stay on near-circular rosettes – they lose angular momentum and move along more radial orbits along the rotating bar: “trapped in the bar”.

Bar drives strong shocks and inflow of gas to the inner regions.

Piner+99 (courtesy J. Stone)



NGC 1512: Barred Galaxy with Starburst Ring



Destroying the bar

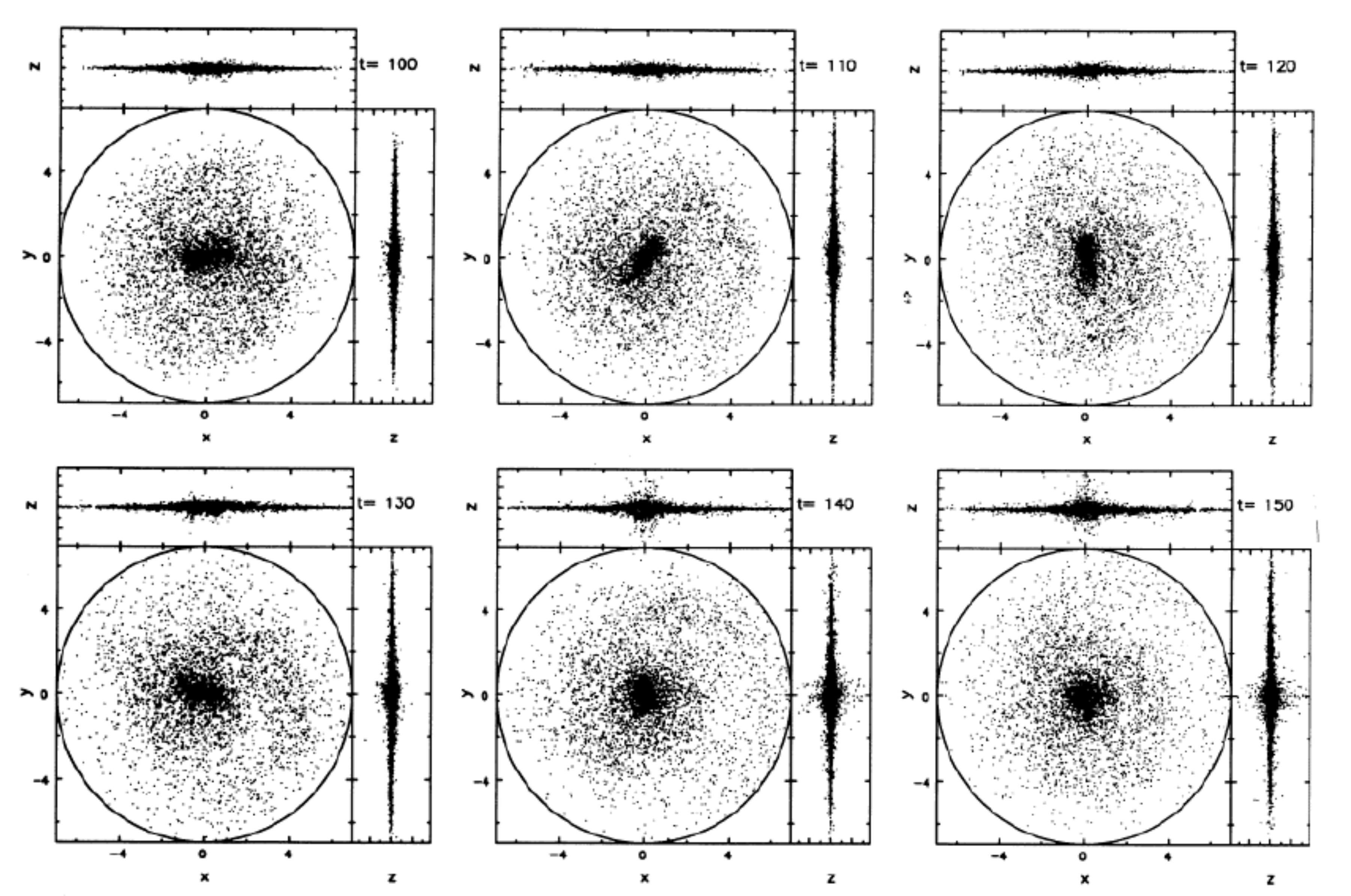
Norman+ 96

To destroy the bar, need to scatter stars off the X_1 orbits that define the bar.

Central mass concentations more massive than ~ few % of disk mass can do this scattering.

What are "central mass concentrations" and how can we get them?

5% mass central concentration grown slowly in barred galaxy simulation. 👉



Disk Stability: the Toomre Q-parameter

Waves can grow or dissipate, depending on kinematics of the rotation curve and the self-gravity of the disk.

Toomre (1964) derived a condition for disk stability for $m = 0$ axisymmetric modes (rings):

σ_R: radial velocity dispersion
κ: epicyclic frequency
Σ: disk mass surface density

$$Q = \frac{\sigma_R \kappa}{3.36 G \Sigma} > 1$$

Qualitatively: *Self-gravity tries to draw a perturbation together, but if over an epicyclic timescale a star skates from one perturbation to another, no single perturbation will grow. ⇒ Stability.*

Used as an indicator for local (small-scale) instabilities:

$Q \gg 1$: "hot disk", very hard to make perturbations grow.
$Q \ll 1$: "cold disk", very unstable, mass perturbations will grow quickly with time.

What happens to a disk with $Q \ll 1$?

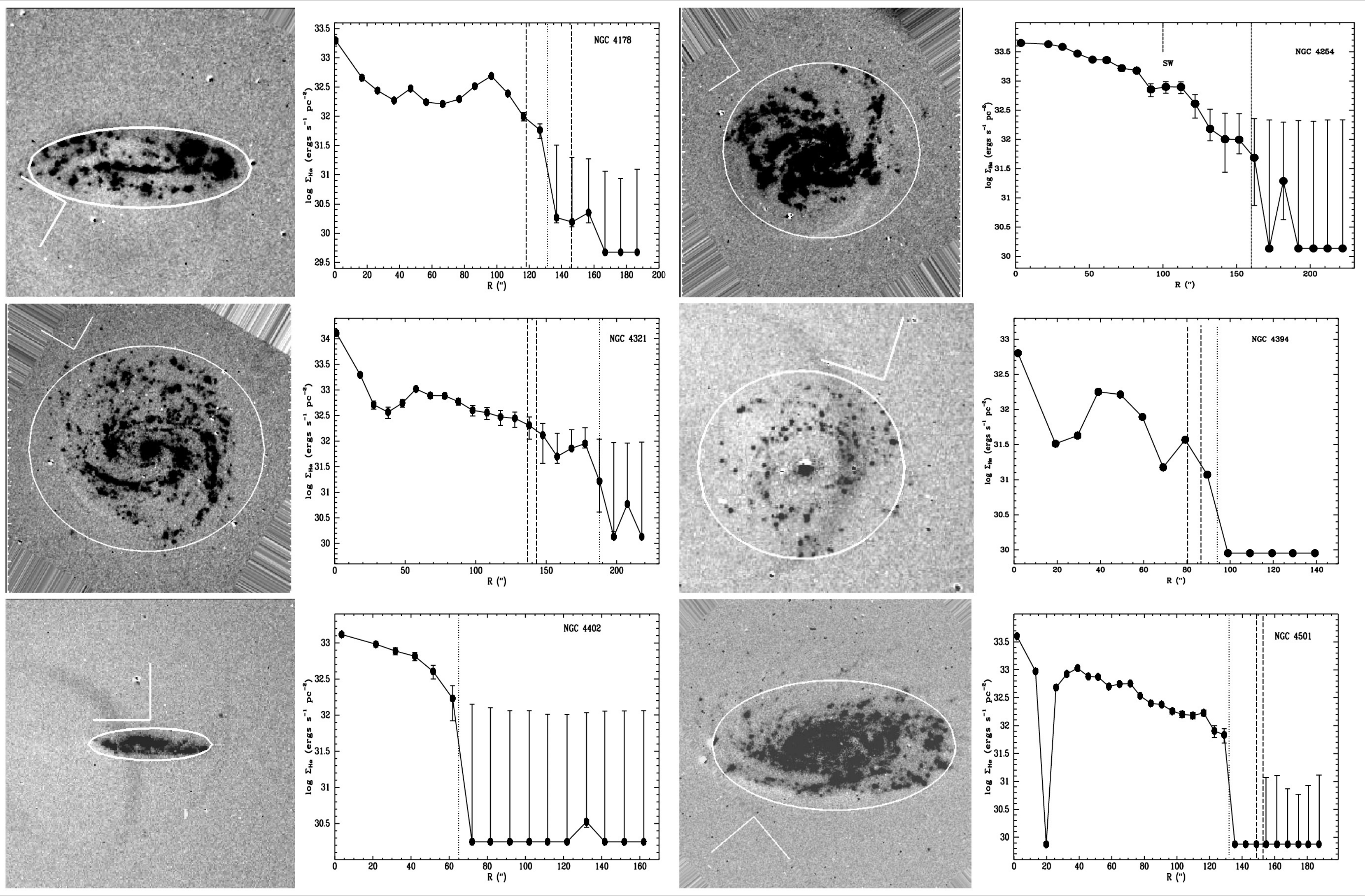
Milky Way (solar neighborhood):

$$\left.\begin{array}{l} \bullet\ \sigma_R \approx 30\ \text{km/s} \\ \bullet\ \kappa \approx 36\ \text{km/s/kpc} \\ \bullet\ \Sigma \approx 50\ \text{M}_\odot/\text{pc}^2 \end{array}\right\} Q \sim 1.4$$

Disk Instabilities and Star Formation

[Martin & Kennicutt 01]

Galaxies often show a reasonably well defined radius beyond which very little widespread star formation is observed.



Hα imaging and Hα surface brightness profiles

Disk Instabilities and Star Formation

Star formation does sometimes occur in galaxy outskirts, but much weaker in intensity and less well organized

"extended disk star formation"
or
"XUV galaxies"

M83
GALEX UV

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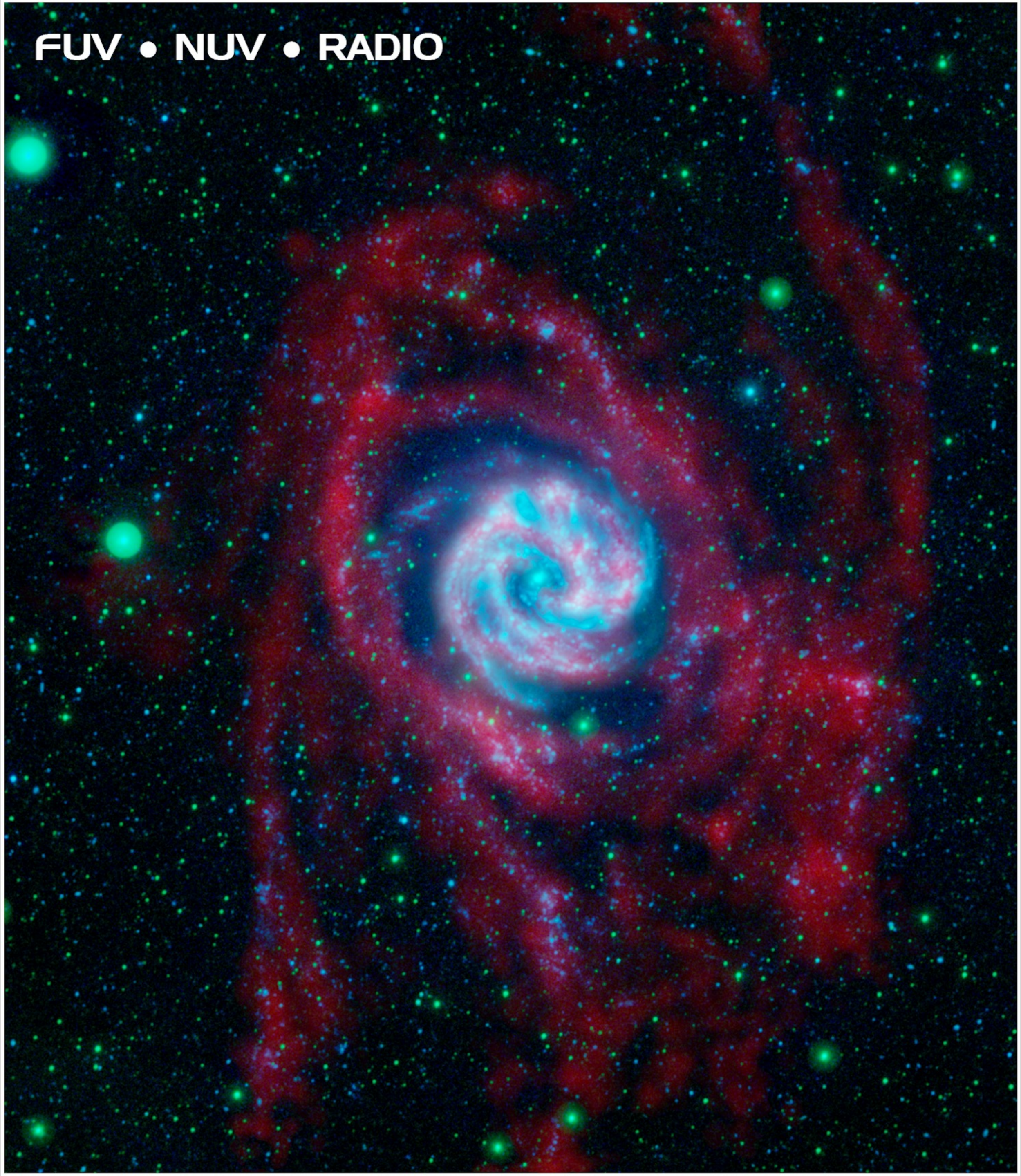
21-cm HI



Instability-Driven Star Formation Scenarios

Remember the expression for local disk stability.

σ_R: radial velocity dispersion
κ: epicyclic frequency
Σ: disk mass surface density

$$Q = \frac{\sigma_R \kappa}{3.36 G \Sigma} > 1$$

At lower densities disks are more stable. Solve for the density where $Q = 1$ and call that the critical density:

$$Q = \frac{\sigma_R \kappa}{3.36 G \Sigma} = 1 \quad \longrightarrow \quad \Sigma_{crit} = \frac{\sigma_R \kappa}{3.36 G}$$

Then we have two regimes:

- High gas density: $\Sigma_{gas} > \Sigma_{crit}$, gas is ”supercritical” and can undergo gravitational collapse to form stars

- Low gas density: $\Sigma_{gas} < \Sigma_{crit}$, gas is stable, not enough gravity to drive collapse. No star formation.

But this is all a simplified theory. Does it actually work?

Does it work?

Maybe?
Kinda?

Outer disks are typically below the critical density, but inner disks are more complicated..

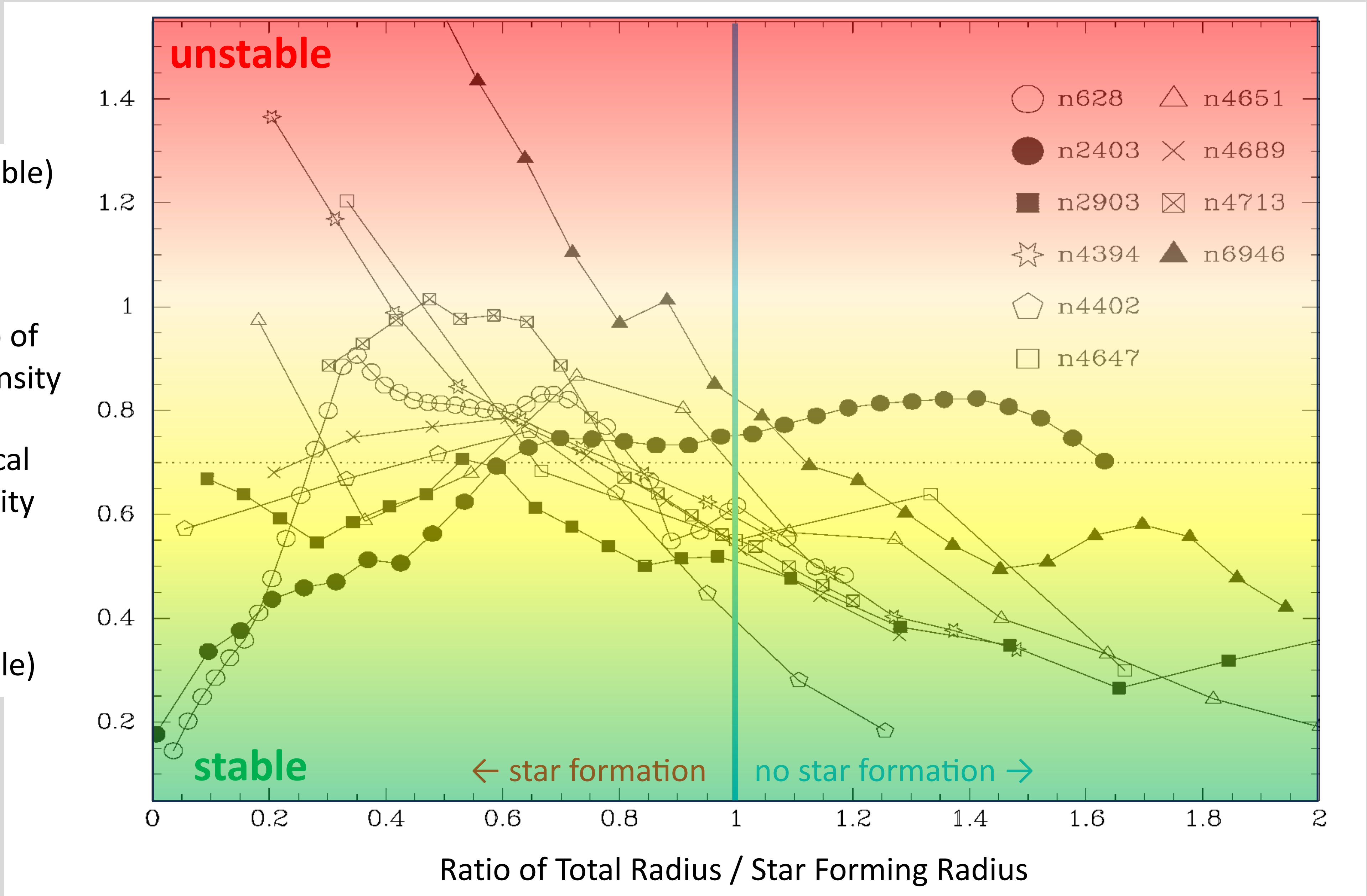
(unstable)

↑

Ratio of gas density to critical density

↓

(stable)

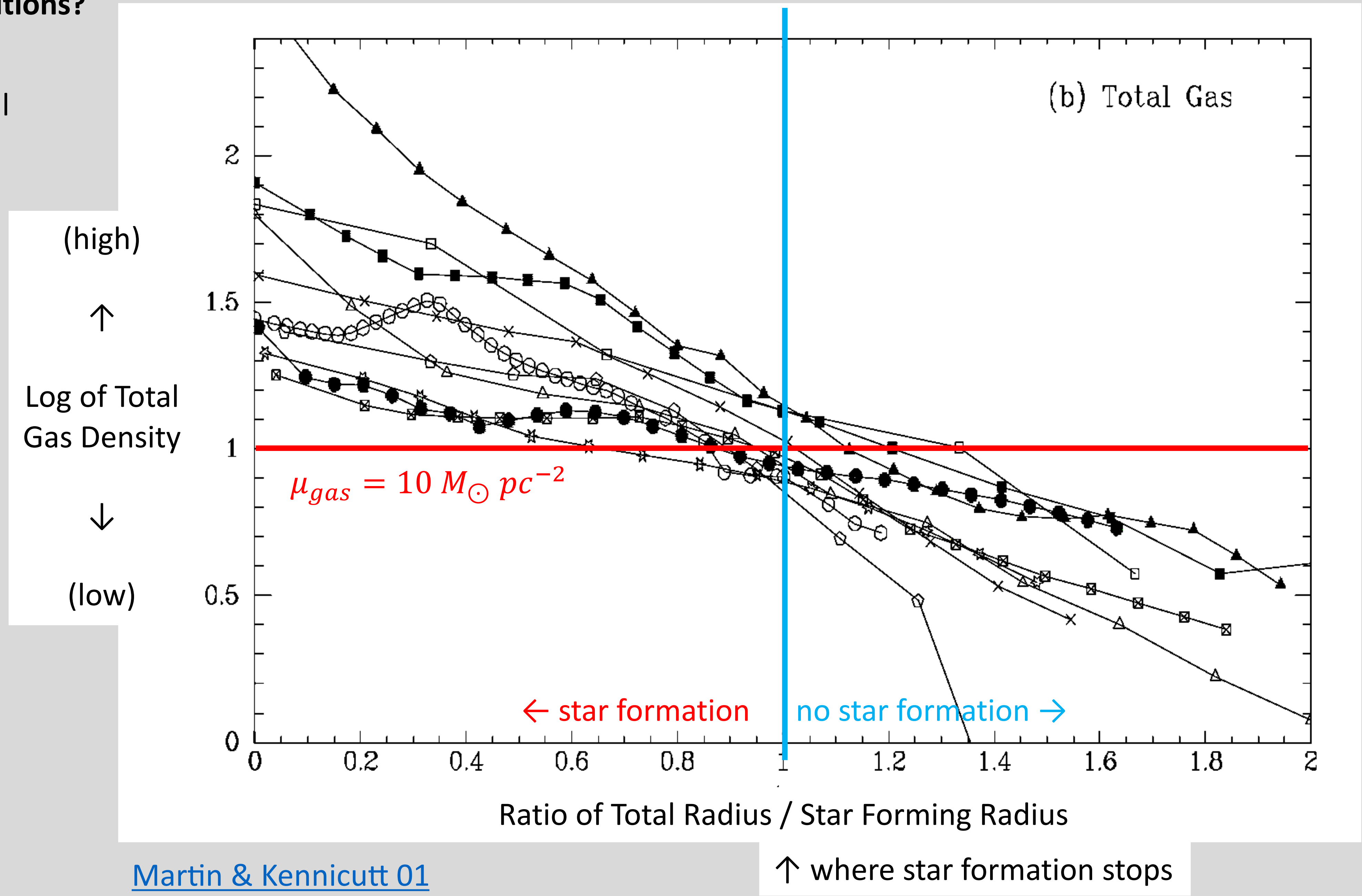


Martin & Kennicutt 01

↑ where star formation stops

Or is it simply local conditions?

Ignore critical/dynamical arguments, just look at total gas density.



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Low density environments

Star formation efficiency greatly reduced.

Is it dynamical stability (a global concept applied locally) or is it purely local conditions? Unclear.

But whatever is happening is connected to disk galaxy evolution: lower SFRs, lower metallicities, more gas-rich, less molecular gas, etc.

Compare:

- LSB galaxies
- low density outskirts of HSB galaxies

Differences and similarities will tell us much....