Image formation from an ideal thin lens

If a lens has a focal length $f_L$, a star on the sky positioned at a distance $\alpha$ from the optical axis will be offset by a distance $s$ in the focal plane where

$$s = f_L \tan(\alpha) = f_L \alpha \text{ in the small angle approx*}$$

So the image plane has a plate scale $\alpha/s = 1/f_L$. This has units of radians/length, so in practical applications, you’ll need to convert to arcsec/mm on the detector.

* remember, when you see a bare angle in a formula, it should be measured in radians, not degrees!
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Figure 5.1. The focusing characteristics of an ideal thin lens. The focal length $f_L$ and the aperture $d$ are each measured in meters. (a) Parallel beam arriving along the lens axis, and focusing at distance $f_L$. (b) Off-axis parallel beam (small angle $\alpha$) also converging at distance $f_L$, but displaced a distance $s = f_L \tan \alpha$ from the lens axis. (c) Extended source subtending angle $\alpha$ and depositing, in a fixed time, energy $E_p \propto (d/f_L)^2$ onto a single pixel of the image plane.
Telescope/Camera “Speed”

For an extended source (galaxy, nebula), the light from the source is deposited over an area that scales as $1/s^2$. So as the plate scale goes up, the energy per pixel on your detector drops. So it takes longer to detect an object.

But if the telescope has a big aperture, it collects a lot of light. Light collecting scales as $D^2$.

Therefore, “speed”, i.e., the total time to “get an exposure” goes as $D^2/s^2 = D^2/fL^2$.

So we can define the focal ratio as $R=f/L/D$, which is written as “f/R”. Then speed=$1/R^2$.

An f/4 telescope is “fast” and a f/16 telescope is “slow”. Faster telescopes are also hard to build!
Telescope Types

Prime focus

Newtonian

Cassegrain

Gregorian

Schmidt-Cassegrain

Schmidt Camera

Maksutov

Refractor
The interaction of light waves with an aperture leads to interference patterns: **diffraction**. In the case of a perfect circular aperture (like an unblocked lens or mirror), this leads to a point source being imaged as an **Airy pattern**:
Diffraction: the Airy pattern

\[ I(\theta) = I_0 \left( \frac{2J_1(ka \sin \theta)}{ka \sin \theta} \right)^2 = I_0 \left( \frac{2J_1(x)}{x} \right)^2 \]

\( k = \text{wavenumber (}= 2\pi / \lambda) \)
\( a = \text{aperture size} \)
\( J_1 \) is a Bessel function
\( \theta = \text{angular radius on image plane} \)

The first minimum of the Airy pattern comes at \( \theta_{\text{min}} = 1.22\lambda / a \)

**Rayleigh limit:**

Once two point sources come closer than the distance to their first Airy minimum, they are not resolved.

Diffraction resolution limit set by wavelength and aperture.
Diffraction limit of telescopes

\( \lambda = 5000 \text{Å (optical)} \)
- \( D = 10 \text{cm} \Rightarrow \theta_{\text{min}} = 1.2 \text{ arcsec} \)
- \( D = 4 \text{m} \Rightarrow \theta_{\text{min}} = 0.03 \text{ arcsec} \)

\( \lambda = 21 \text{cm (radio)} \)
- \( D = 20 \text{m} \Rightarrow \theta_{\text{min}} = 35 \text{arcmin} \)
- \( a = 100 \text{m} \Rightarrow \theta_{\text{min}} = 7 \text{ arcmin} \)

*to make a perfect mirror, the surface must be polished to accuracy < \( \lambda \)*