Filters and Magnitudes
Filters

Kron B
WTMM-1

\[ \lambda_{\text{Central}} = 4340.15 \text{Å} \]
\[ \lambda_{\text{FWHM}} = 1051.13 \text{Å} \]
\[ T_{\text{max}} = 73.45\% \text{ at } 4360.00 \text{Å} \]

Dispersion: 5.00Å/Interval

Transmission, %

Wavelength, Angstroms
Filters

Filters don’t always look this regular! ➔

Defining a filter (a reasonable attempt)

Mean wavelength: \( \lambda_c = \frac{\int \lambda T(\lambda) d\lambda}{\int T(\lambda) d\lambda} \)

Filter width:

- \( \Delta \lambda = \text{FWHM} \) (works if filter transmission curve is reasonably square/symmetric)
- \( \Delta \lambda = \text{width of the equivalent square filter at } T=100\% \) giving same throughput.

Filters are not perfectly repeatable, due to manufacturing differences, environment differences, etc. One person’s B filter is slightly different from another’s.

To standardize photometry, filter corrections are typically needed, and they depend on the color of what you are observing. So, for example, \( m_B = m_{B,\text{obs}} + C(B-V) \) where \( C \) is called the “color term” of the individual filter.
Choosing filters

trying to measure physical features of an astronomical spectrum.

For example, Johnson broadband filters.
Filter systems

<table>
<thead>
<tr>
<th>System</th>
<th>Filter</th>
<th>$\lambda_0$</th>
<th>$\Delta\lambda_{1/2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$UBV$ (Johnson-Morgan)</td>
<td>$U$</td>
<td>3650 Å</td>
<td>700 Å</td>
</tr>
<tr>
<td></td>
<td>$B$</td>
<td>4400 Å</td>
<td>1000 Å</td>
</tr>
<tr>
<td></td>
<td>$V$</td>
<td>5500 Å</td>
<td>900 Å</td>
</tr>
<tr>
<td>Six-color (Stebbins-Whitford-Kron)</td>
<td>$U$</td>
<td>3550 Å</td>
<td>500 Å</td>
</tr>
<tr>
<td></td>
<td>$V$</td>
<td>4200 Å</td>
<td>800 Å</td>
</tr>
<tr>
<td></td>
<td>$B$</td>
<td>4900 Å</td>
<td>800 Å</td>
</tr>
<tr>
<td></td>
<td>$G$</td>
<td>5700 Å</td>
<td>800 Å</td>
</tr>
<tr>
<td></td>
<td>$R$</td>
<td>7200 Å</td>
<td>1800 Å</td>
</tr>
<tr>
<td></td>
<td>$I$</td>
<td>10,300 Å</td>
<td>1800 Å</td>
</tr>
<tr>
<td>Infrared (Johnson)</td>
<td>$R$</td>
<td>7000 Å</td>
<td>2200 Å</td>
</tr>
<tr>
<td></td>
<td>$I$</td>
<td>8800 Å</td>
<td>2400 Å</td>
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<tr>
<td></td>
<td>$J$</td>
<td>1.25 $\mu$m</td>
<td>0.38 $\mu$m</td>
</tr>
<tr>
<td></td>
<td>$K$</td>
<td>2.2 $\mu$m</td>
<td>0.48 $\mu$m</td>
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<tr>
<td></td>
<td>$L$</td>
<td>3.4 $\mu$m</td>
<td>0.70 $\mu$m</td>
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<tr>
<td></td>
<td>$M$</td>
<td>5.0 $\mu$m</td>
<td>1.2 $\mu$m</td>
</tr>
<tr>
<td></td>
<td>$N$</td>
<td>10.4 $\mu$m</td>
<td>5.7 $\mu$m</td>
</tr>
<tr>
<td>$uvby\beta$ (Strömgren-Crawford)</td>
<td>$u$</td>
<td>3500 Å</td>
<td>340 Å</td>
</tr>
<tr>
<td></td>
<td>$v$</td>
<td>4100 Å</td>
<td>200 Å</td>
</tr>
<tr>
<td></td>
<td>$b$</td>
<td>4700 Å</td>
<td>160 Å</td>
</tr>
<tr>
<td></td>
<td>$y$</td>
<td>5500 Å</td>
<td>240 Å</td>
</tr>
<tr>
<td></td>
<td>$\beta$</td>
<td>4860 Å</td>
<td>30 Å, 150 Å</td>
</tr>
</tbody>
</table>
Filter systems

Bessell ARAA 2005
Filter systems

example of “similar but different” filters:

SDSS ugriz vs CFHT ugriz
Flux through a filter

An astronomical object emits a spectrum given by $f_{\lambda}$ (in erg/s/cm$^2$/Å). This is referred to as spectral flux density (flux per wavelength).

*Note: Flux density is often also written in terms of frequency: $f_{\nu}$ (in erg/s/cm$^2$/Hz)*

The total flux (in erg/s/cm$^2$) passing through the filter is given by

$$f = \int f_{\lambda} \times T_F(\lambda) \, d\lambda$$

- $f_{\lambda}$: star spectrum
- $T_F(\lambda)$: filter transmission
- $f_{\lambda} \times T_F(\lambda)$: spectrum through filter
- $\int f_{\lambda} \times T_F(\lambda) \, d\lambda$: flux through filter
Magnitudes as a measure of flux

If fluxes (f) are in erg/s/cm$^2$, magnitudes of different things measured in the same filter are related by

$$m_1 - m_2 = -2.5 \log_{10}(f_1/f_2)$$

Magnitudes are defined relative to some standard flux or object.

- $\Delta m = 1$ mag $\Rightarrow$ factor of 2.512 in flux
- $\Delta m = 5$ mag $\Rightarrow$ factor of 100 in flux

Using differential calculus, if the uncertainties ($\sigma$) are small you can show that

$$\sigma_m = -1.086 \left(\frac{\sigma_f}{f}\right) \approx \left(\frac{\sigma_f}{f}\right)$$

In other words, for small uncertainties the uncertainty in magnitudes is the fractional uncertainty in flux.
Magnitudes as a measure of distance

\[ m - M = 5 \log_{10}(d) - 5 \]

\( m - M \) = “distance modulus”
\( d \) = distance \textbf{in parsecs}

M87 at a distance of \( \approx 16 \) Mpc has a distance modulus of

\[ m - M = 5 \log(16 \times 10^6) - 5 = 31.02 \text{ mags} \]

Again using differential calculus, if the uncertainties (\( \sigma \)) are small you can show that

\[ \frac{\sigma d}{d} \approx 0.5\sigma_{(m-M)} \]

\textit{In other words, the fractional uncertainty in distance is about half the uncertainty in the distance modulus.}
Surface Brightness

If magnitude is defined by \( m = -2.5 \log f + C \), we can define **surface brightness** as flux per unit area (A) on the sky:

\[ \mu = -2.5 \log(f/A) + C \]

or

\[ \mu = -2.5 \log f + 2.5 \log A + C \]

So

\[ \mu = m + 2.5 \log A \]

if area is measured in arcsec\(^2\), then surface brightness (\( \mu \)) is given in mag/arcsec\(^2\)

However, units not withstanding, surface brightnesses (just like magnitudes) are not additive. **In other words** \( m \neq \mu \times A \) !!!
Surface Brightness

Surface brightness is **distance independent**, an intrinsic property of the object being studied (at least until you get to cosmological distances).

Therefore an *observable surface brightness* (in mag/arcsec$^2$) corresponds to an *intrinsic luminosity (surface) density*.

For example, $\mu_B = 27.0$ mag/arcsec$^2$ corresponds to $\approx 1 \, L_{B,\odot}/pc^2$. 

![M101](image1.png)  
![Malin 1](image2.png)
Colors

Color \equiv m_{\lambda_1} - m_{\lambda_2}, \text{ so for example } B - V = m_B - m_V

If \( m = -2.5 \log f + C \), then

Color \equiv -2.5 \log f_{\lambda_1} + C_{\lambda_1} - (-2.5 \log f_{\lambda_2} + C_{\lambda_2}), \text{ or }

Color \equiv -2.5 \log(f_{\lambda_1} / f_{\lambda_2}) + (C_{\lambda_1} - C_{\lambda_2})

Important points:
• Like magnitudes, colors are measured relative to some reference object
• Convention is to always list the bluer filter first. B–V, not V–B.
• This means that smaller (and into more negative) numbers are bluer colors.

Look at color-color plot:
Magnitude Systems

\[ m_\lambda = -2.5 \log f + C_\lambda \]

Conceptually, the zeropoint (C) can either be based on physical units or on a reference star. See [Bessell (ARAA) 05](https://www.journals.uchicago.edu/doi/10.1086/316873) for review.

Don’t confuse magnitude systems with filter systems! – M Bershady

The Vega System

By definition, Vega (α Lyr): \( m = 0.00 \) at all wavelengths. Therefore Vega has a color of 0.00 in all colors by definition.

Therefore, in the Vega system, a color of 0.0 is NOT the same as equal flux at all wavelengths (a so-called “flat spectrum”). Magnitudes measure brightness relative to Vega and colors measure colors relative to Vega.
Magnitude Systems

![Vega Spectrum 11/07/2013](image)

- H-delta: 410.17 nm
- H-gamma: 434.05 nm
- H-beta: 486.13 nm
- H-alpha: 656.28 nm
- Tel O2: 686.90 nm
- Tel O2: 760.50 nm

(Courtesy KSU Astronomy)
Magnitude Systems: the AB and STMAG systems

**Physical Units:** flux = Energy/area/time = erg/s/cm$^2$

*(where cm$^2$ refers to the area of your light collector)*

We can define the **monochromatic flux density** as

\[ f_\nu = \text{Energy/area/time/frequency} = \text{erg/s/cm}^2/\text{Hz} \]

*(1 Jansky = $10^{-23} = \text{erg/s/cm}^2/\text{Hz})*

or

\[ f_\lambda = \text{Energy/area/time/wavelength} = \text{erg/s/cm}^2/\AA \]

So there are two monochromatic magnitude systems:

**AB system:** $m_\nu$ or $m_{\text{AB}} = -2.5\log(f_\nu) - 48.6$  
*(SDSS mags are [close to] AB)*  
(where a spectrum with constant $f_\nu$ gives colors=0)

and

**STMAG system:** $m_\lambda = -2.5\log(f_\lambda) - 21.1$  
(where a spectrum with constant $f_\lambda$ gives colors=0)

\[ f_\nu \text{ and } f_\lambda \text{ related by } v f_\nu = \lambda f_\lambda \]

*constant $f_\nu$ is not the same as constant $f_\lambda$!*
Comparing Magnitude Systems

Relating \( f_\nu \) and \( f_\lambda \):

Since \( \nu f_\nu = \lambda f_\lambda \),

\[
f_\nu = \frac{\lambda}{\nu} f_\lambda = \frac{\lambda^2}{c} f_\lambda
\]

If we think of the average flux density across a filter bandpass, the pivot wavelength for the filter is defined such that:

\[
\langle f_\nu \rangle = \frac{\lambda_p^2}{c} \langle f_\lambda \rangle
\]

Note units! Why do we care about photon flux? Because CCD detectors count photons, not energy!
Worked Example: Vega in different units (see spectral_conversion.ipynb)

for an A0 star with \(V=0.0\) (this is Vega!), the monochromatic flux density at 5492Å is

\[ f_\lambda = 3.63 \times 10^{-9} \text{ erg/s/cm}^2/\text{Å} \]

Note: \(V\) is another way of writing \(m_V\) -- but never \(M_V\)!

which can also be written in terms of frequency:

\[ f_\nu = \left(\frac{\lambda^2}{c}\right)f_\lambda = 3.65 \times 10^{-20} \text{ erg/s/cm}^2/\text{Hz} = 3650 \text{ Jy} \]

or AB magnitudes:

\[ m_{AB} = -2.5 \log(f_\nu) - 48.6 = -0.006 \]

to convert to photon flux, divide by \(f_\lambda\) by the photon energy \((h\nu)\):

\[ f_\lambda \ [\text{photons}] \approx 1000 \text{ photons/s/cm}^2/\text{Å} \]

and if the V filter has a bandpass of \(~900\ \text{Å}\), the total photon flux through a V filter is about 900,000 photons/s/cm².

Remember: these are all “top of the atmosphere” values, i.e., airmass \(X=0\).