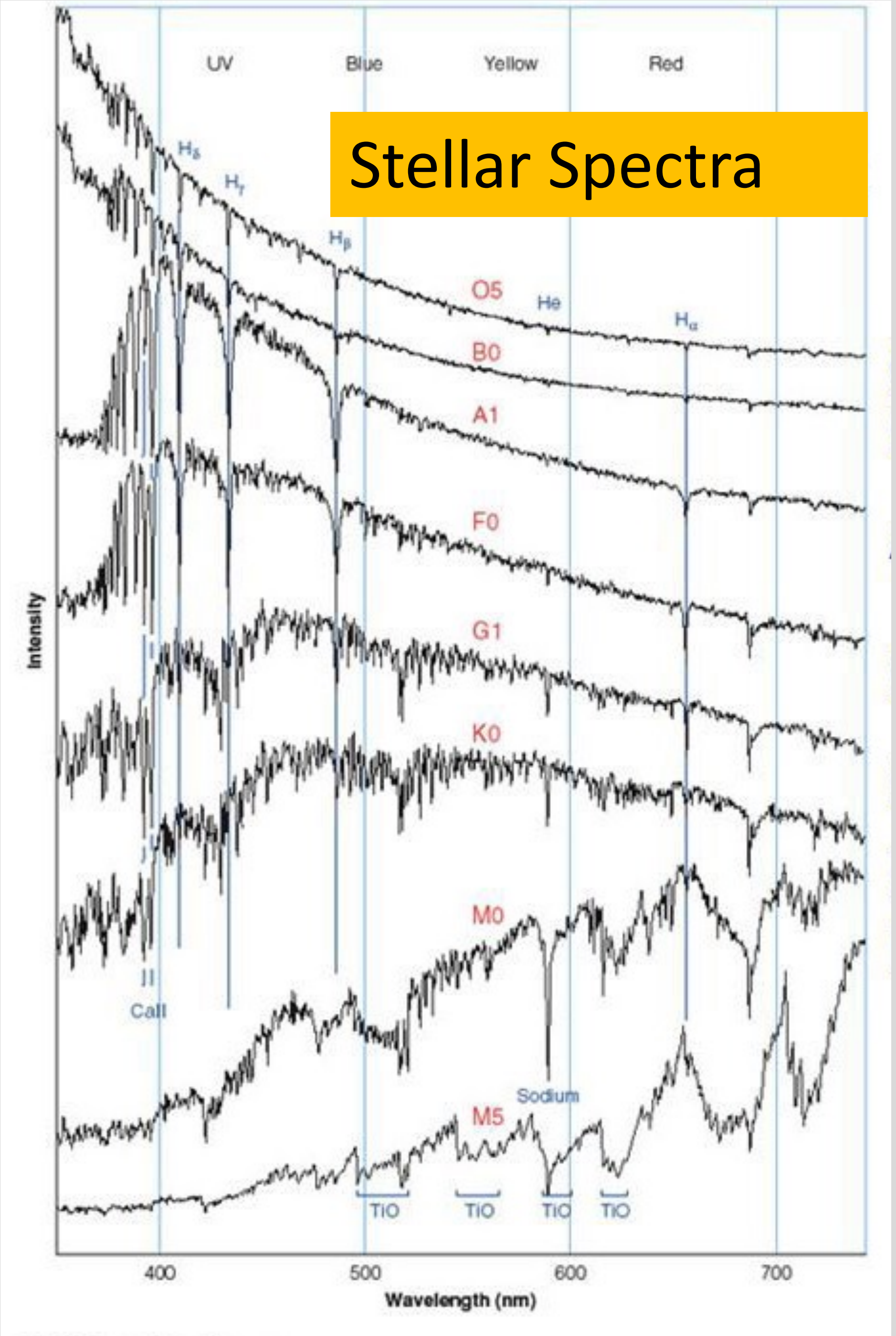
Spectroscopy: Disperse (spread) the light of an object out into its spectrum.

- **Measure spectral lines:** chemical abundances, temperatures, ionization source, density, etc.
- **Measure redshifts**.
- **Measure velocities**.



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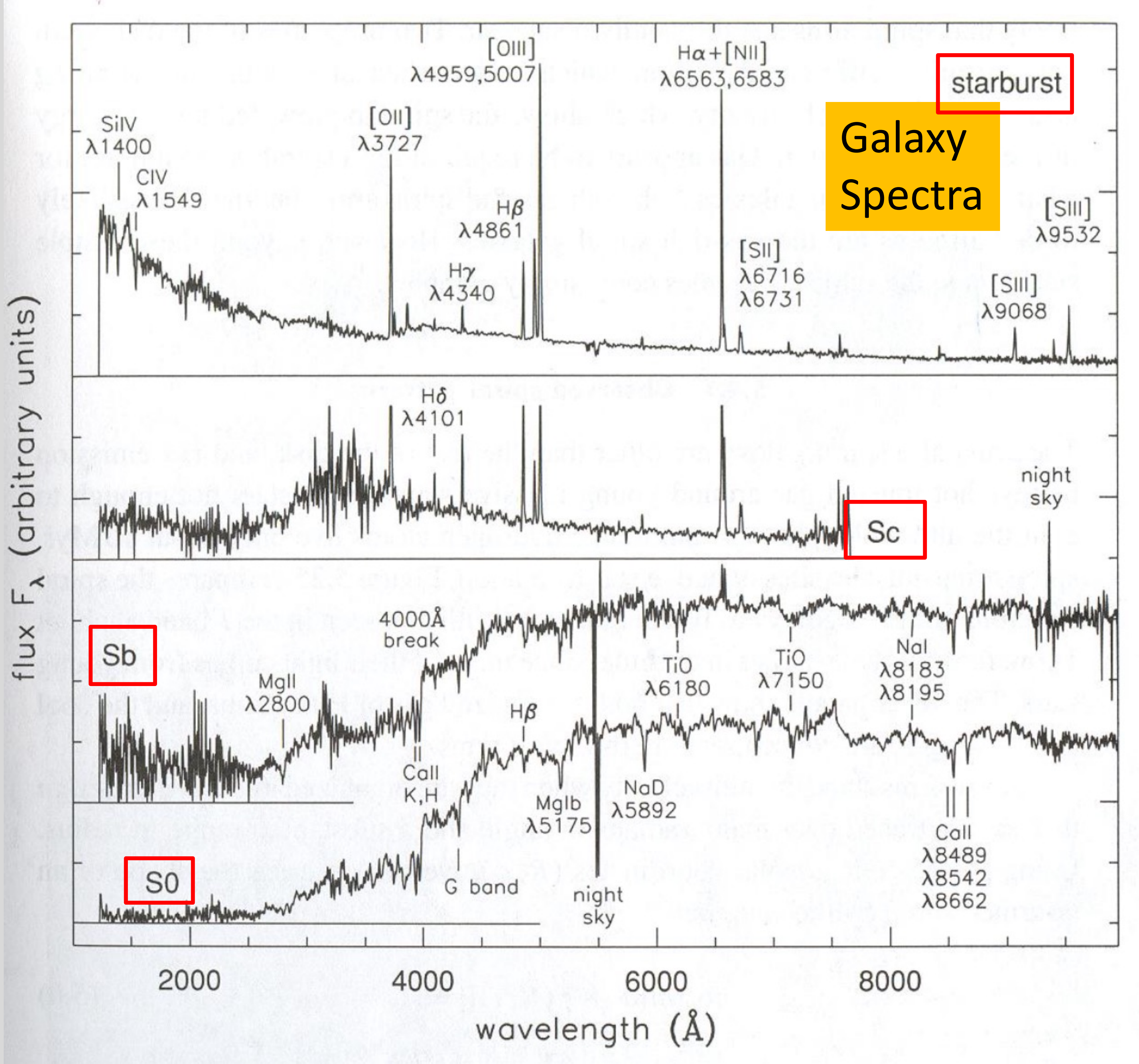
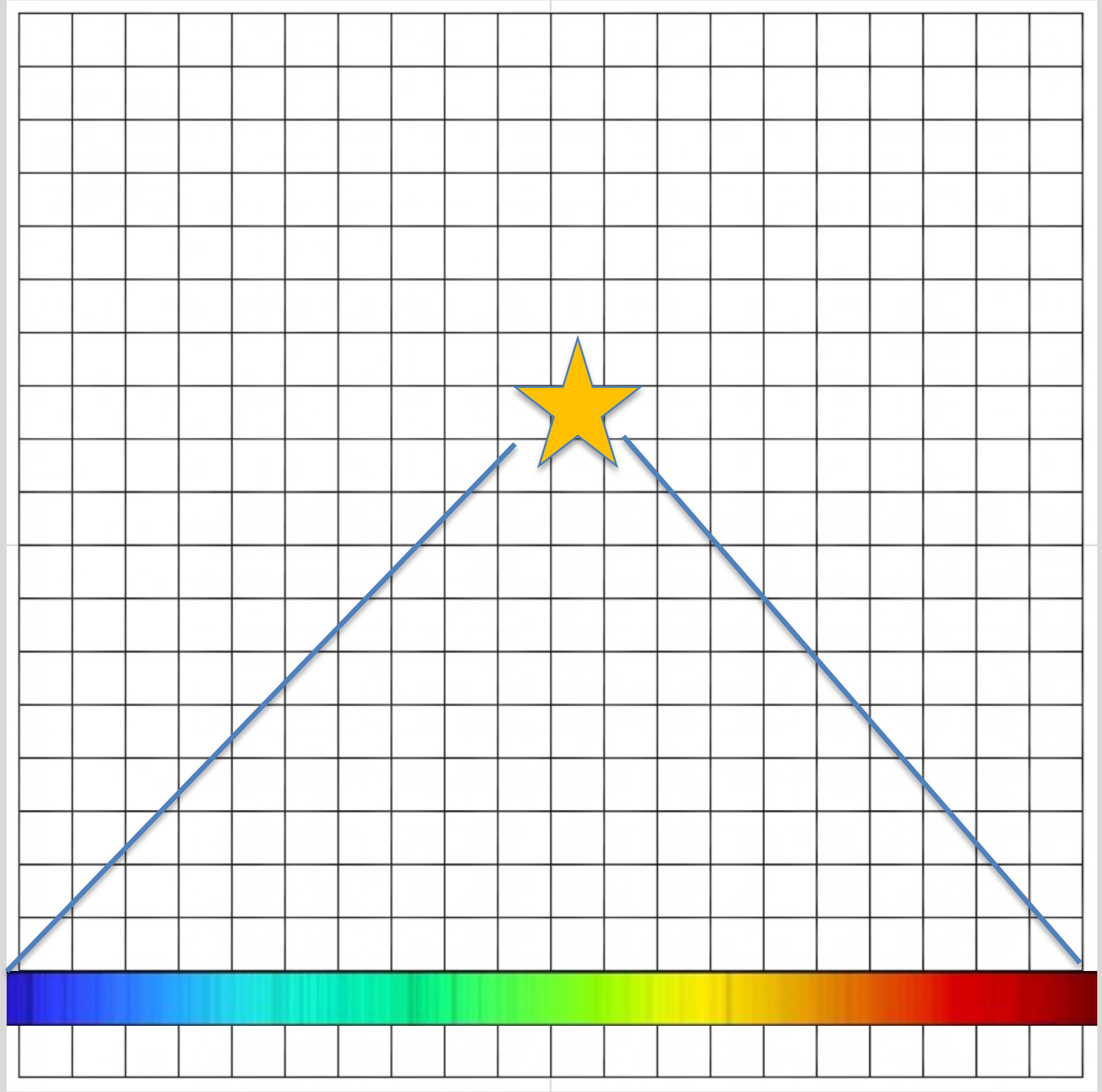


Imaging vs Spectroscopy

Imaging: Take all the light from a star, dump it into a few pixels. Easy to detect even relatively faint sources.

Spectroscopy: Spread the light out in wavelength, dumping it across a whole long swath of pixels. Many fewer photons per pixel, much fainter to measure.

Faint object spectroscopy: the domain of large telescopes.



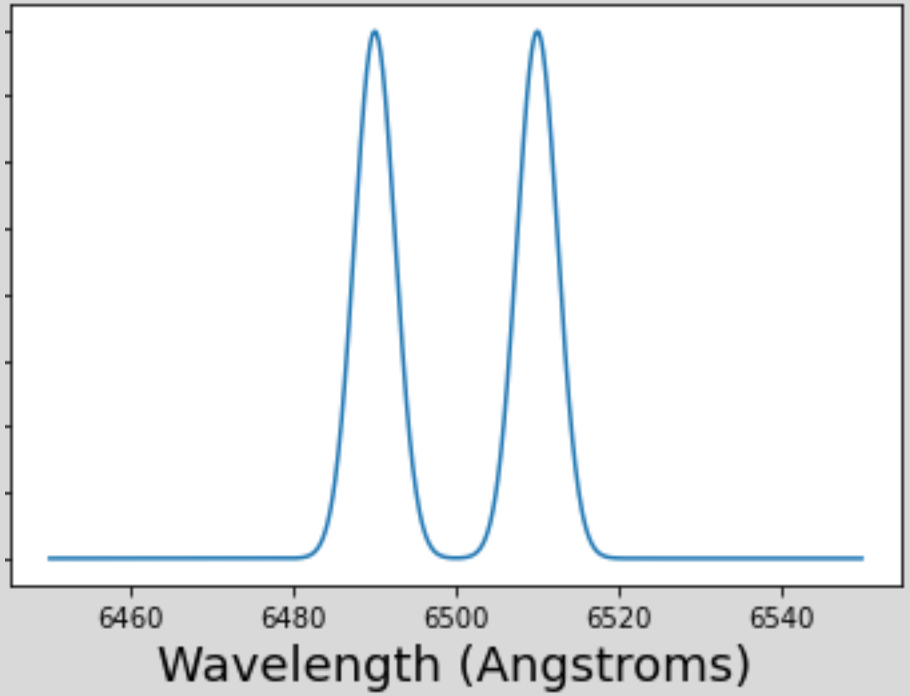
Spectral Resolution

Typically characterized by

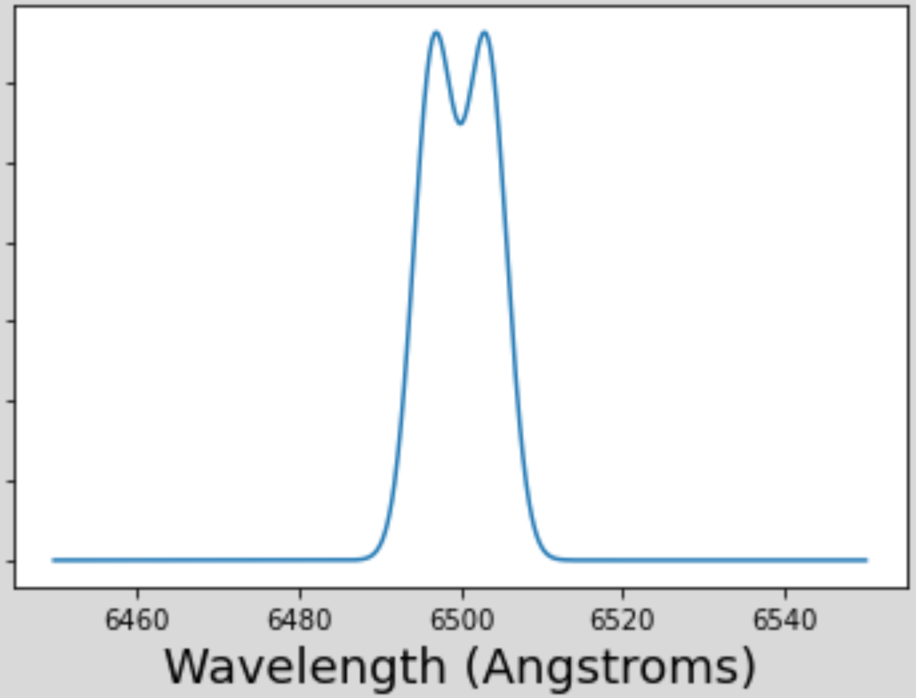
$$R = \lambda/\Delta\lambda$$

where Δλ is the wavelength difference of two spectral lines that can just be distinguished separately.

These two lines at $\lambda = 6490\ \text{Å}, 6510\ \text{Å}$ ($\Delta\lambda = 20\text{Å}$) are easily distinguishable



These two lines at $\lambda = 6496.75\ \text{Å}, 6503.25\ \text{Å}$ ($\Delta\lambda = 6.5\ \text{Å}$) are barely distinguishable.
So this spectrum has a resolution $R \approx 1000$.



Spectral Resolution

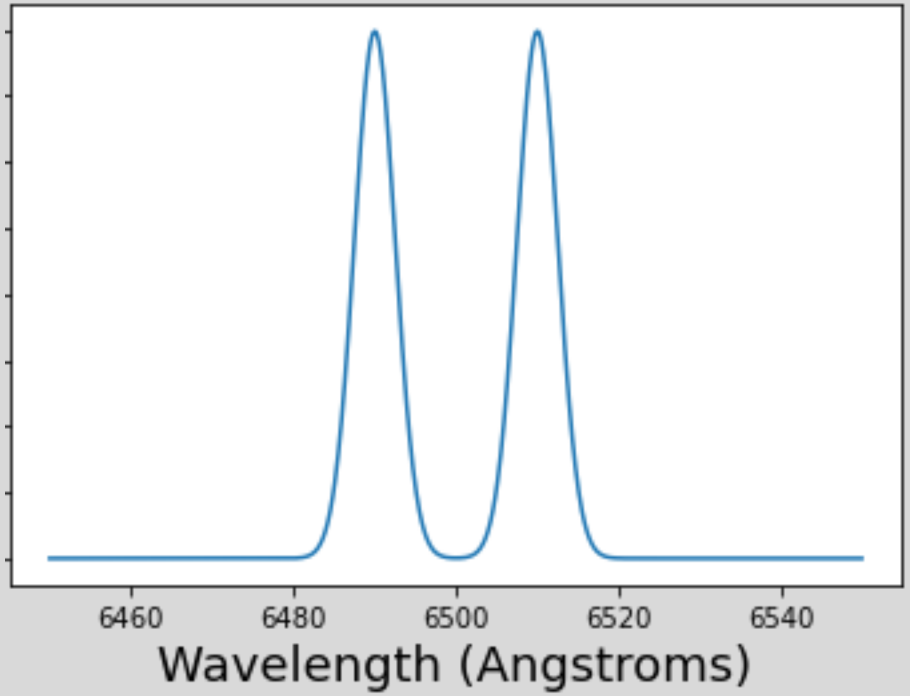
Typically characterized by

$$R = \lambda/\Delta\lambda \; (= \underbrace{c/\Delta v}_{\text{Doppler velocity}})$$

At Hα ($\lambda = 6563$ Å),
$\Delta\lambda = 6.5$ Å $\Rightarrow$ $\Delta v = 300$ km/s

where Δλ is the wavelength difference of two spectral lines that can just be distinguished separately.

These two lines at $\lambda = 6490$ Å, 6510 Å ($\Delta\lambda = 20$Å) are easily distinguishable



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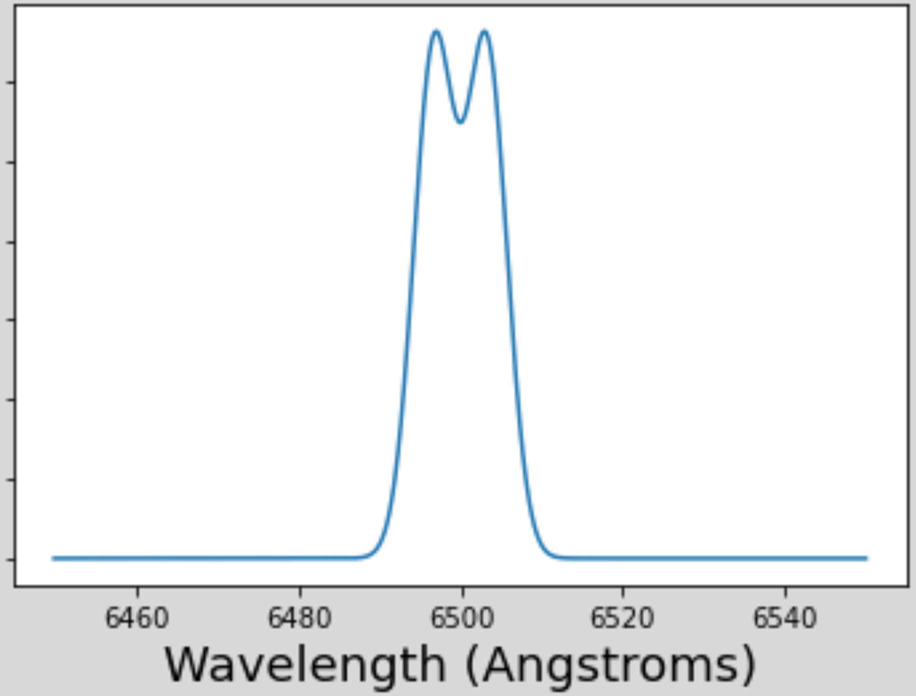


Low Resolution (R ~ 600)

At Hα (λ=6563Å):

- $\Delta\lambda = 6.5$Å
- $\Delta v = 500$ km/s

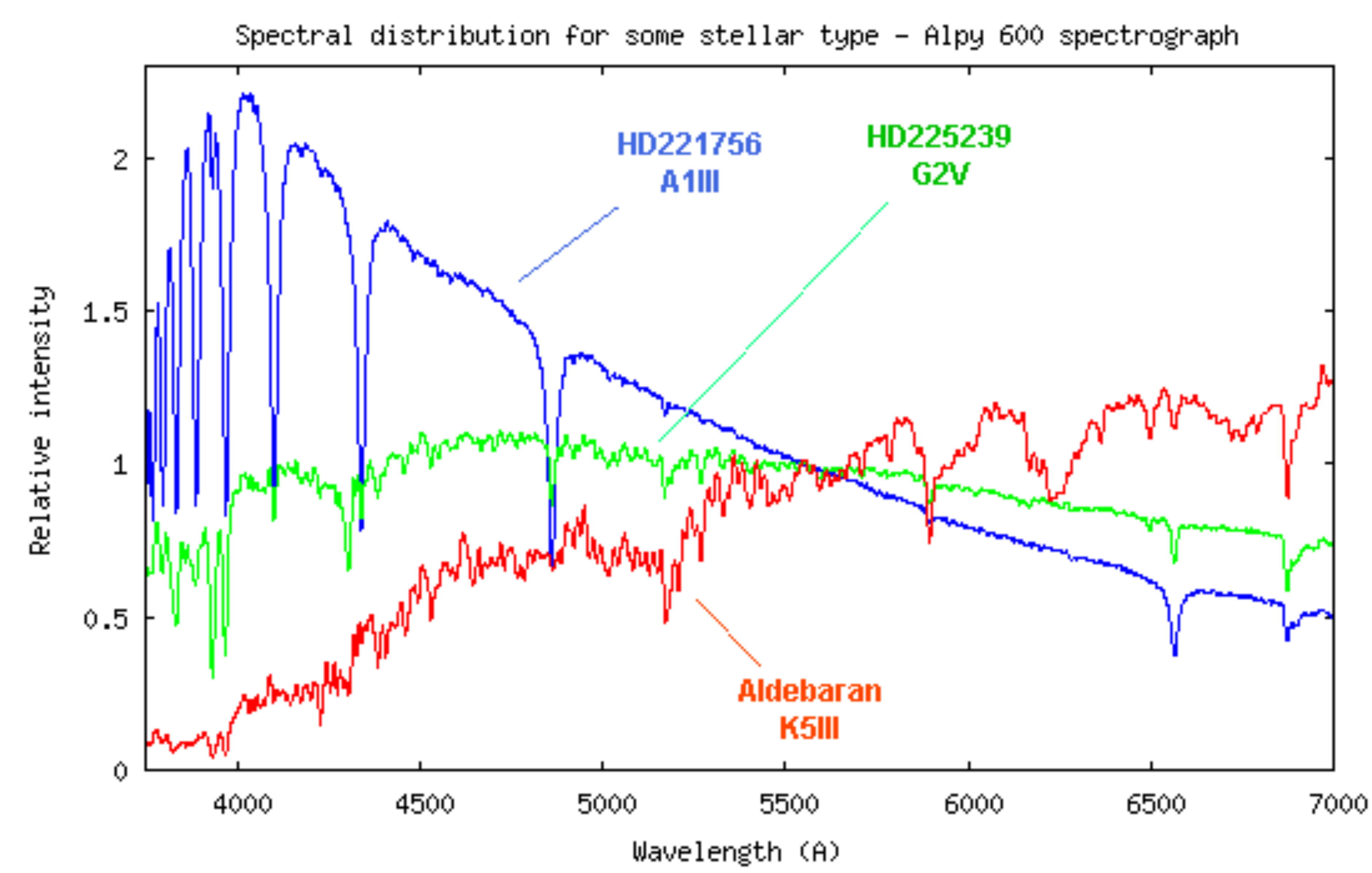
Very broad spectral range

Shows continuum shape

Identifies strong lines

Weak lines are indistinguishable

Gives very crude velocity or redshift information.



Medium Resolution (R ~ 2000)

At Hα (λ=6563Å):

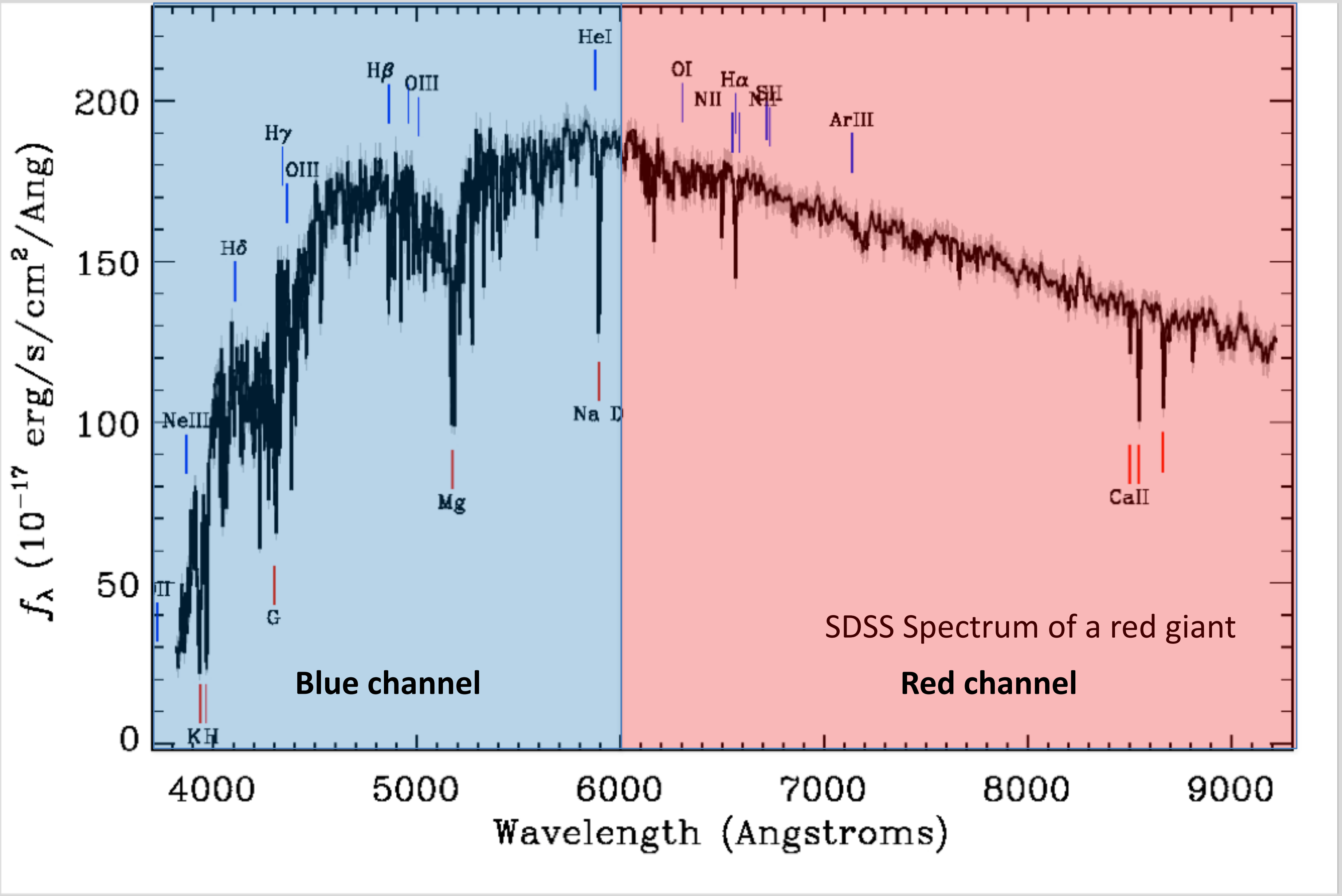
- Δλ = 3 Å
- Δv = 150 km/s

Narrower spectral range

Shows continuum shape

Identifies strong lines and moderately weak lines

Gives better velocity or redshift information.



High Resolution (R ~ 50,000)

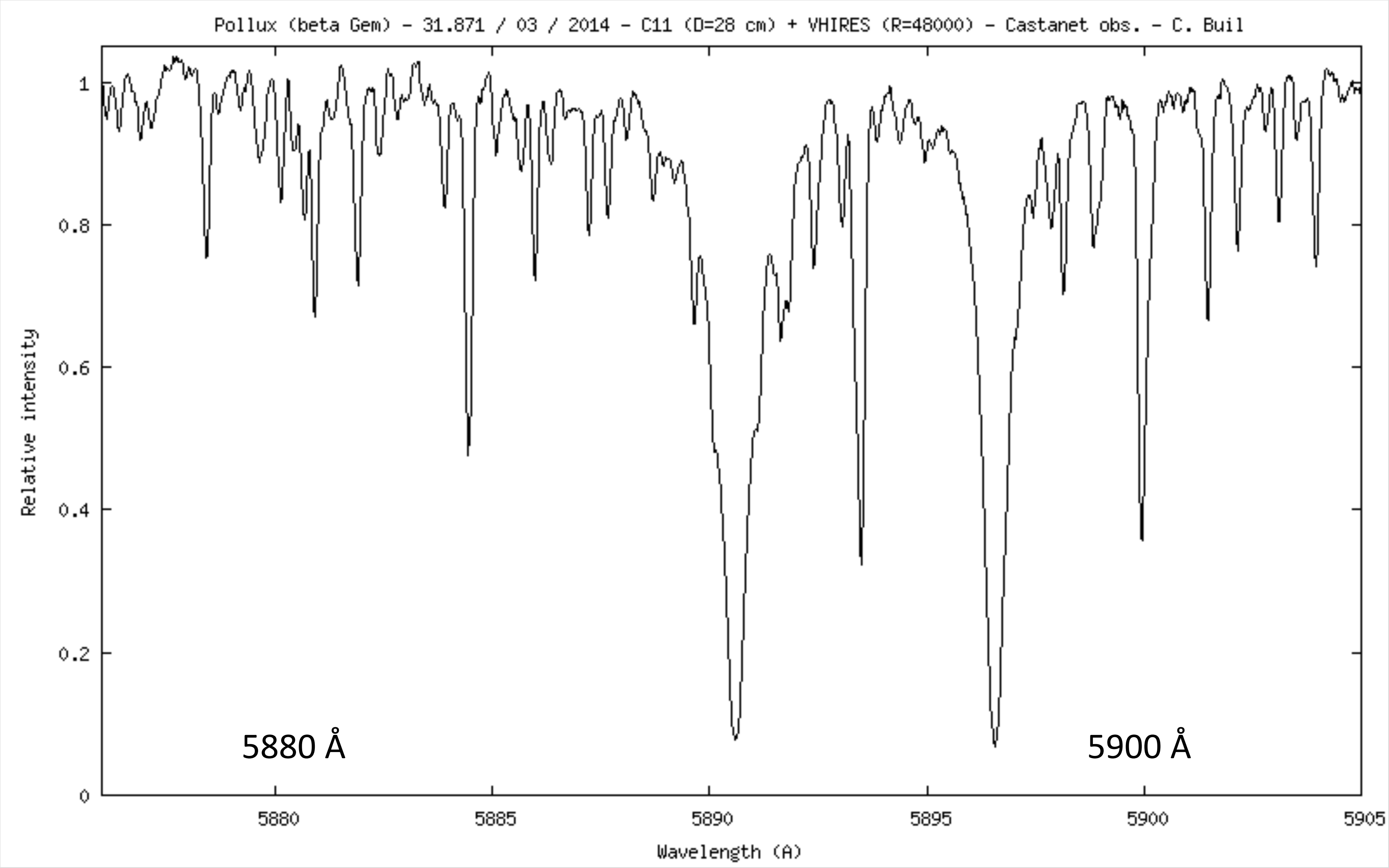
At Hα (λ=6563Å):
- Δλ = 0.13 Å
- Δv = 6 km/s

Very narrow wavelength range.

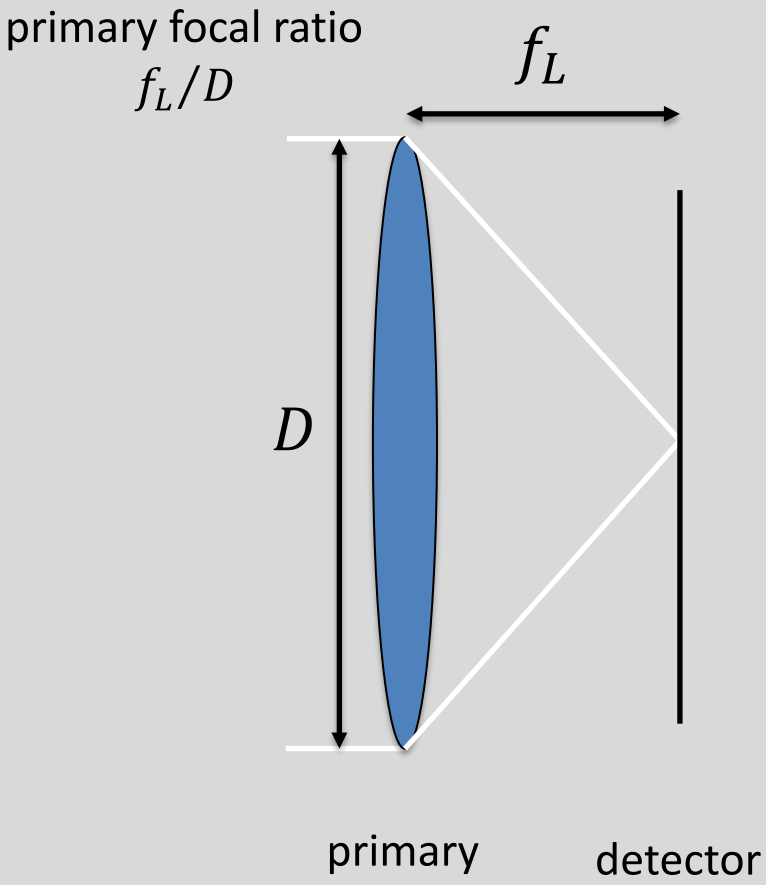
Identifies many weak lines

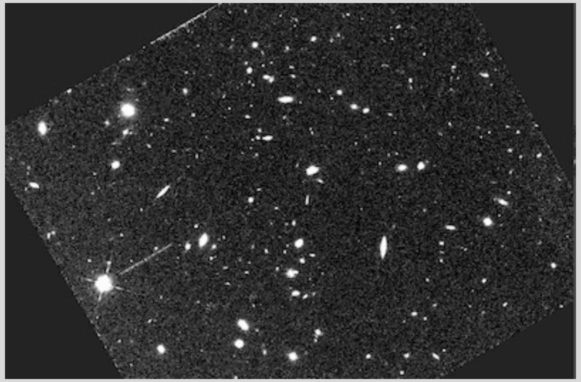
Gives exquisite velocity information.

But because the light is so dispersed in wavelength, the source needs to be very bright or the telescope very big.

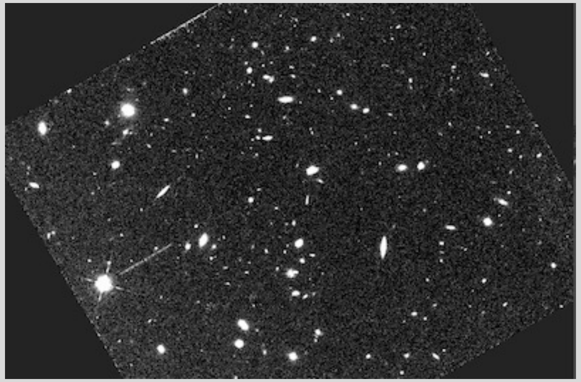


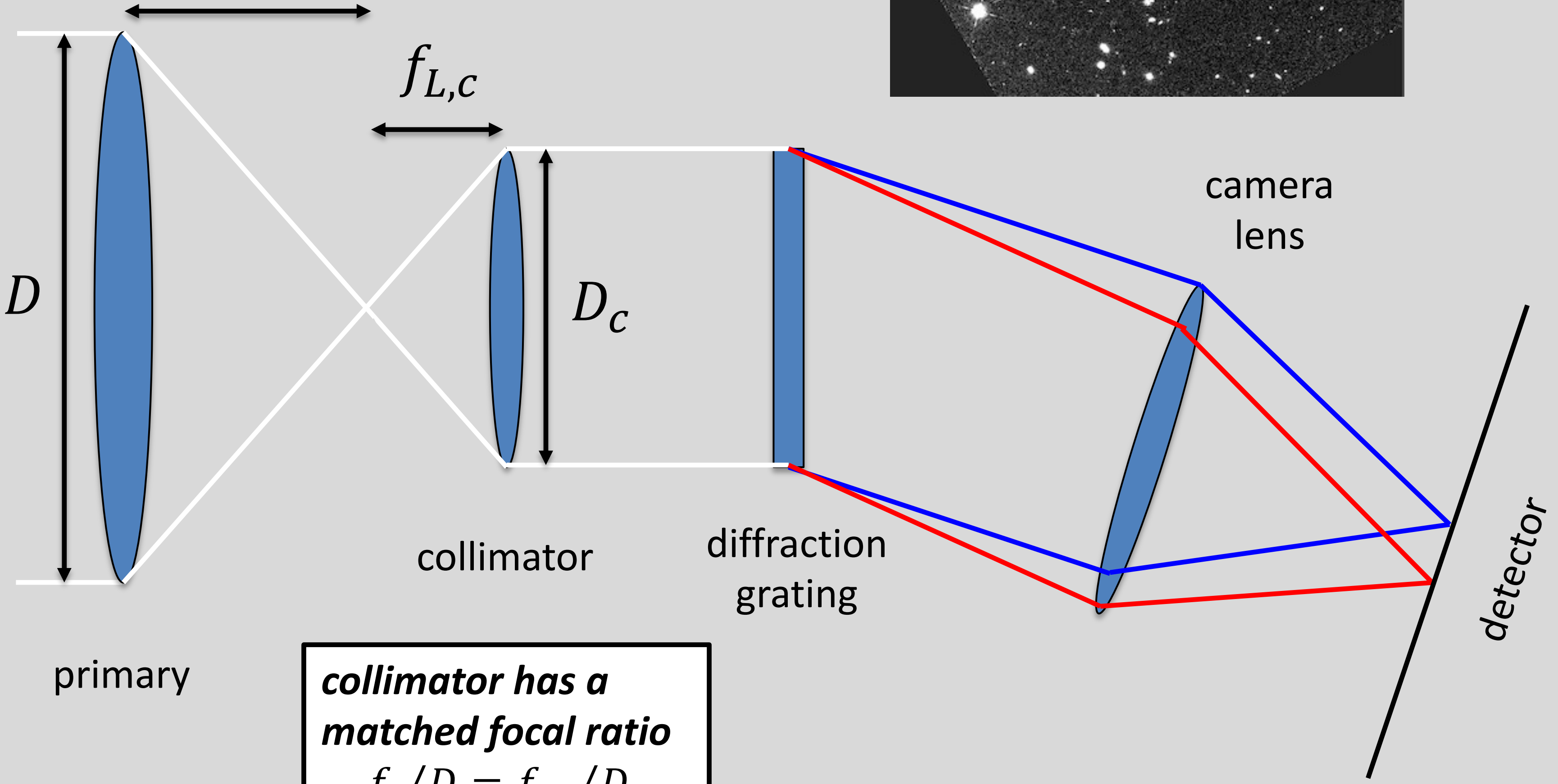
Simple Imaging System





Simple Spectrograph

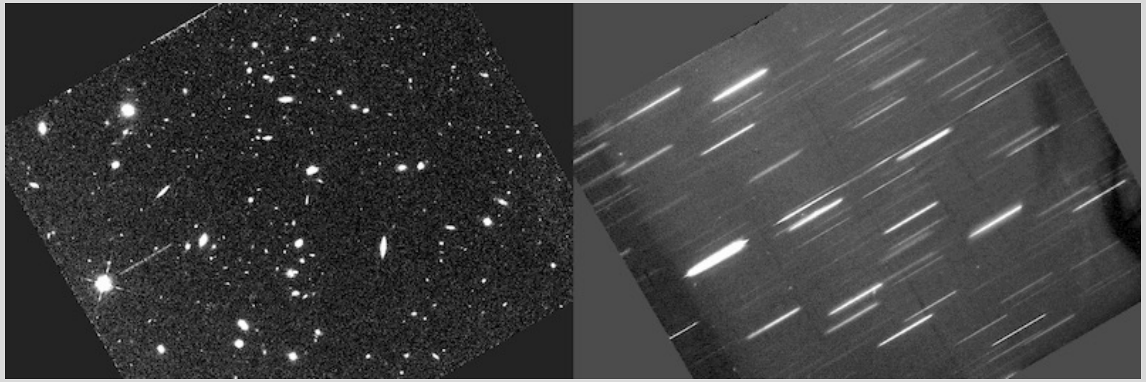


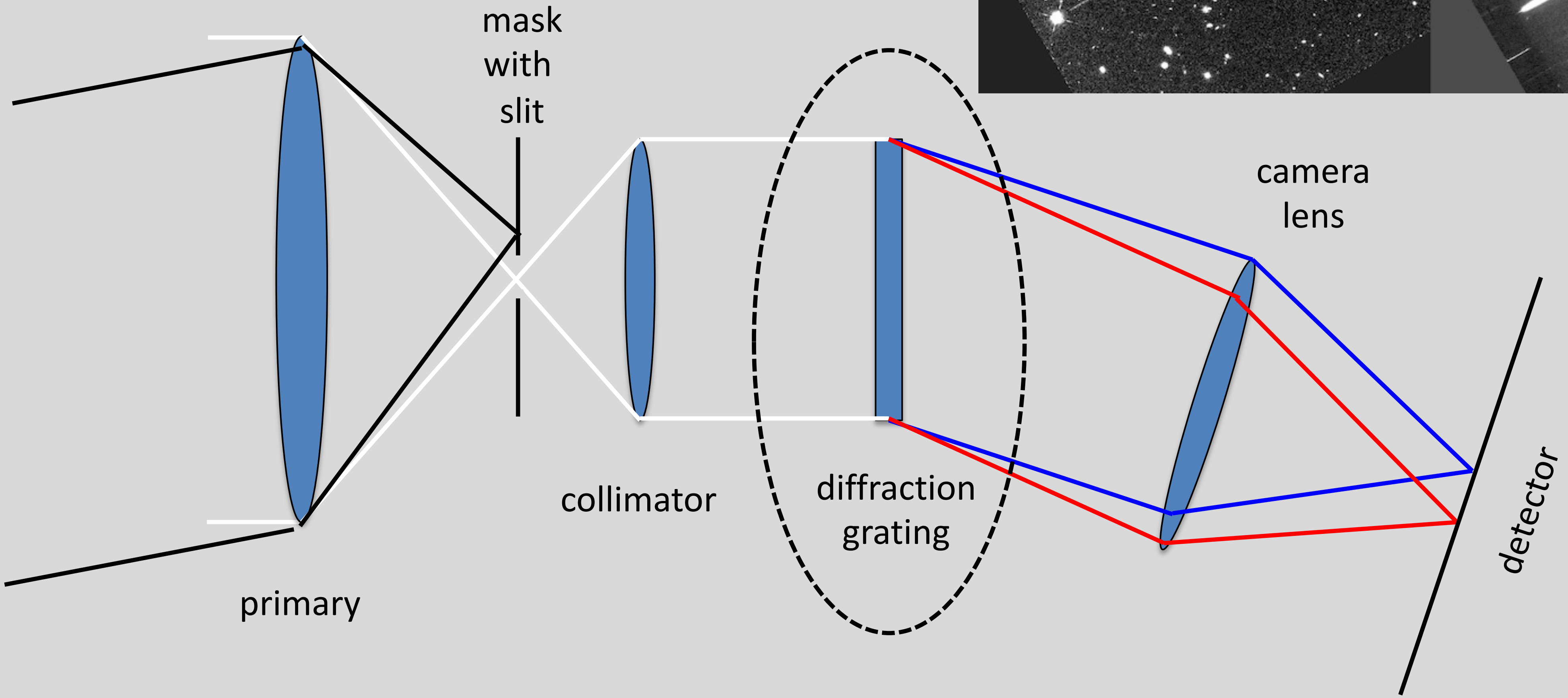


collimator has a matched focal ratio
$f_L/D = f_{L,c}/D_c$

(For ease of sketching, this shows a transmissive system: refracting telescope, transmission grating. Most telescopes and spectrographs actually use a reflecting system.)

Simple Spectrograph





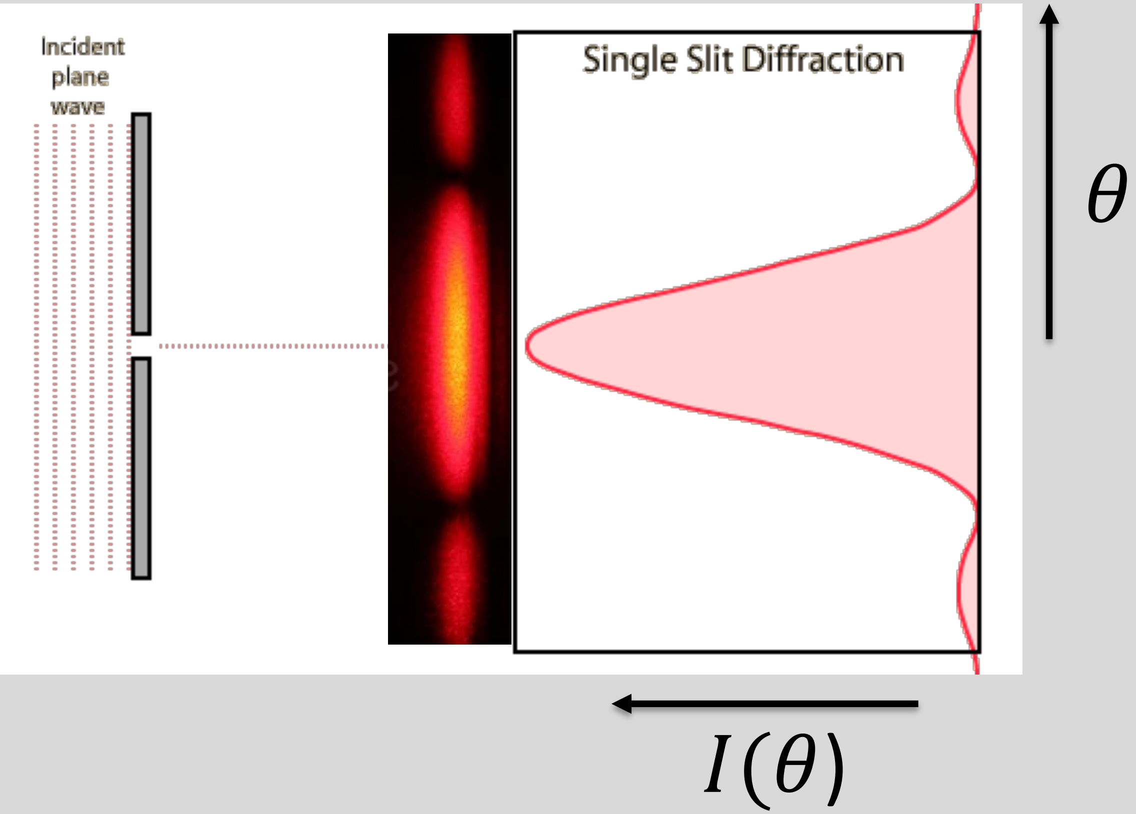
Spectrograph slit: Insert a mask in front of the spectrograph with a small slit that only lets through light from the object you want to take a spectrum of.

Single Slit Diffraction

Remember PHYS 122?

*We're **not** talking about the spectrograph slit here!*

First think about ***diffraction***. Pass a wave through a single slit aperture and you'll get a diffraction pattern:



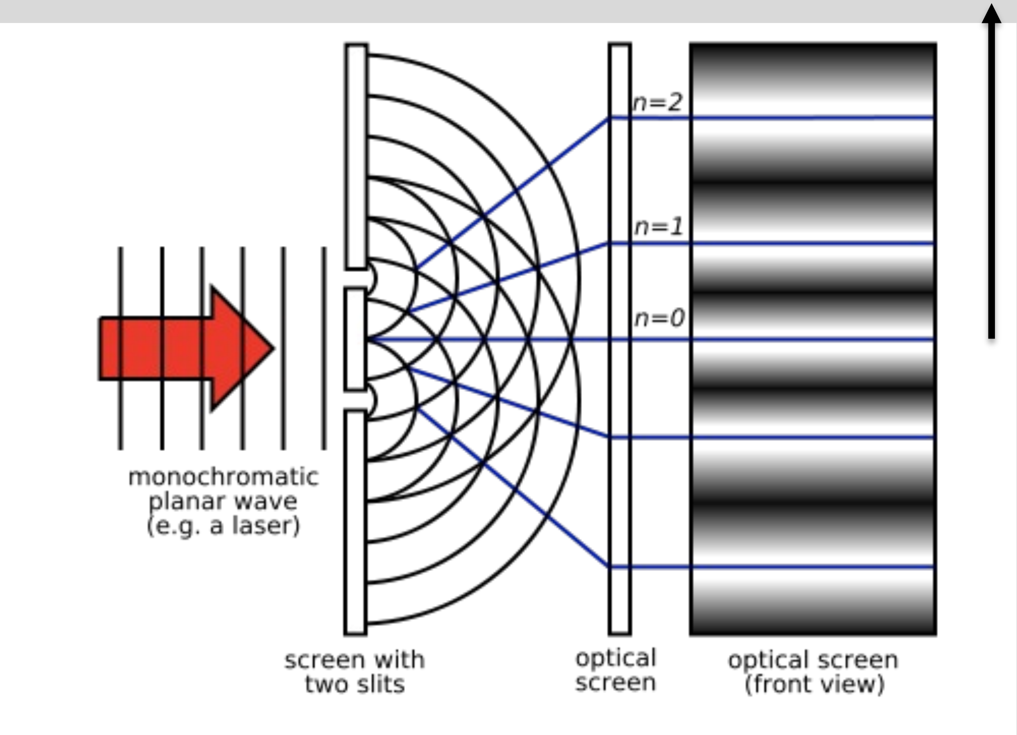
$$I(\theta) = I_0 \frac{\sin^2(\pi\alpha)}{(\pi\alpha)^2}$$

$$\alpha = \frac{a \sin\theta}{\lambda}$$

θ = projected angle from center of peak
a = slit width
λ = wavelength

Two Slit Interference

Now think about two-slit ***interference*** *(again from PHYS 122….)*:



θ

$$I(\theta) = I_0 \cos^2(\pi\delta)$$

$$\delta = \frac{d \sin\theta}{\lambda}$$

θ = projected angle from center of peak
d = distance between slits
λ = wavelength

Condition for intensity peaks:
δ = integer (m, "order")
or
$\sin\theta = m\lambda/d$

Peaks are separated by λ/d

Combined Two-slit diffraction and interference pattern

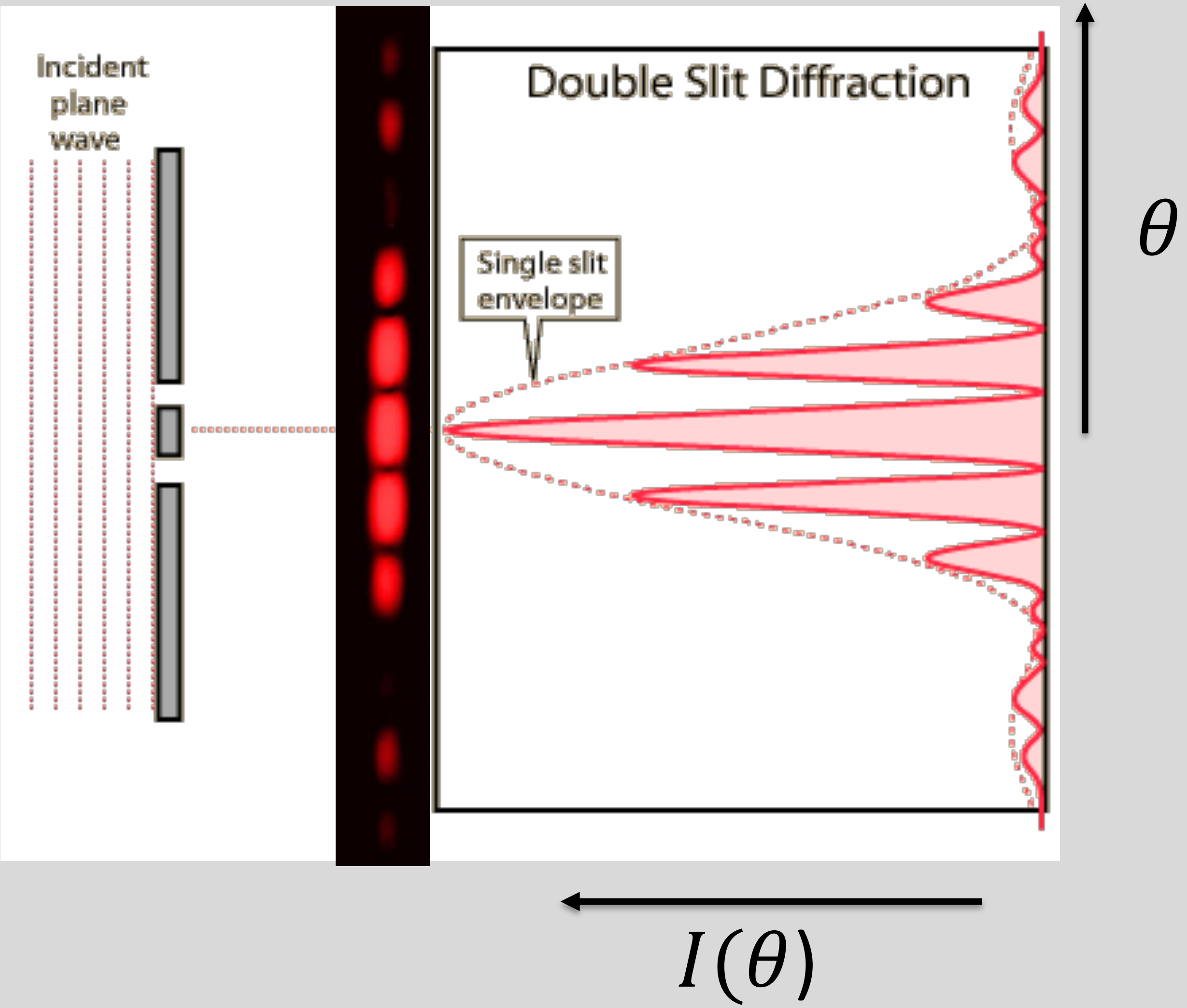
Combine diffraction and interference:

$$I(\theta) = I_0 \overbrace{\frac{\sin^2(\pi\alpha)}{(\pi\alpha)^2}}^{\text{single-slit diffraction}} \times \overbrace{\cos^2(\pi\delta)}^{\text{two-slit interference}}$$

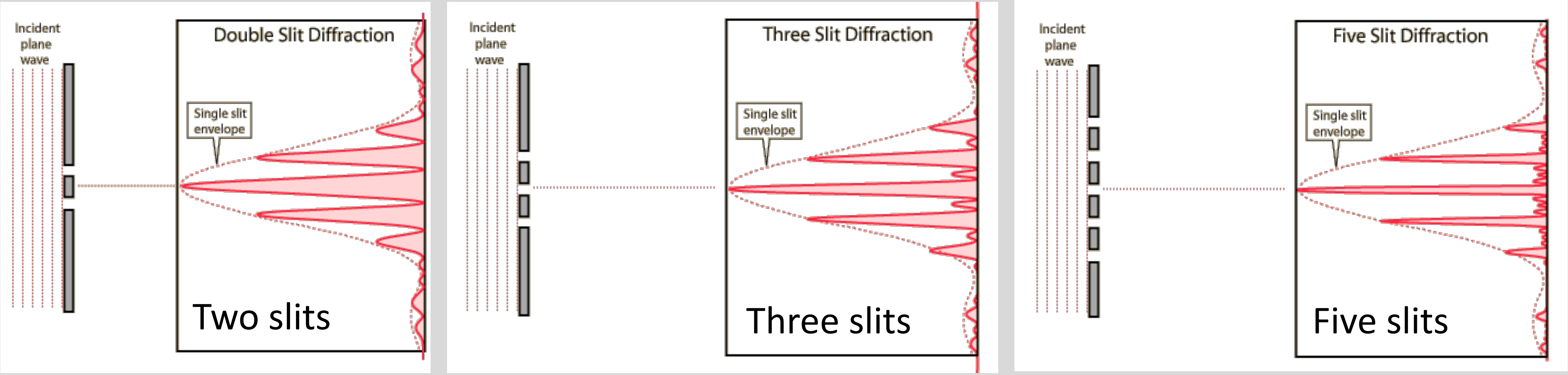
$$\alpha = \frac{a\sin\theta}{\lambda}$$

$$\delta = \frac{d\sin\theta}{\lambda}$$

θ = projected angle from center of peak
a = slit width
d = distance between slits
λ = wavelength



Multi-slit interference: Keep adding slits with same width and spacing.



Math more complicated, but as the number of slits (N) increases, subsequent peaks get narrower.

Spacing of maxima: λ/d
Width of peaks: $\lambda/(Nd)$

d = distance between slits
λ = wavelength

Important: Spacing/width of peaks is a function of slit separation (d), not slit width!

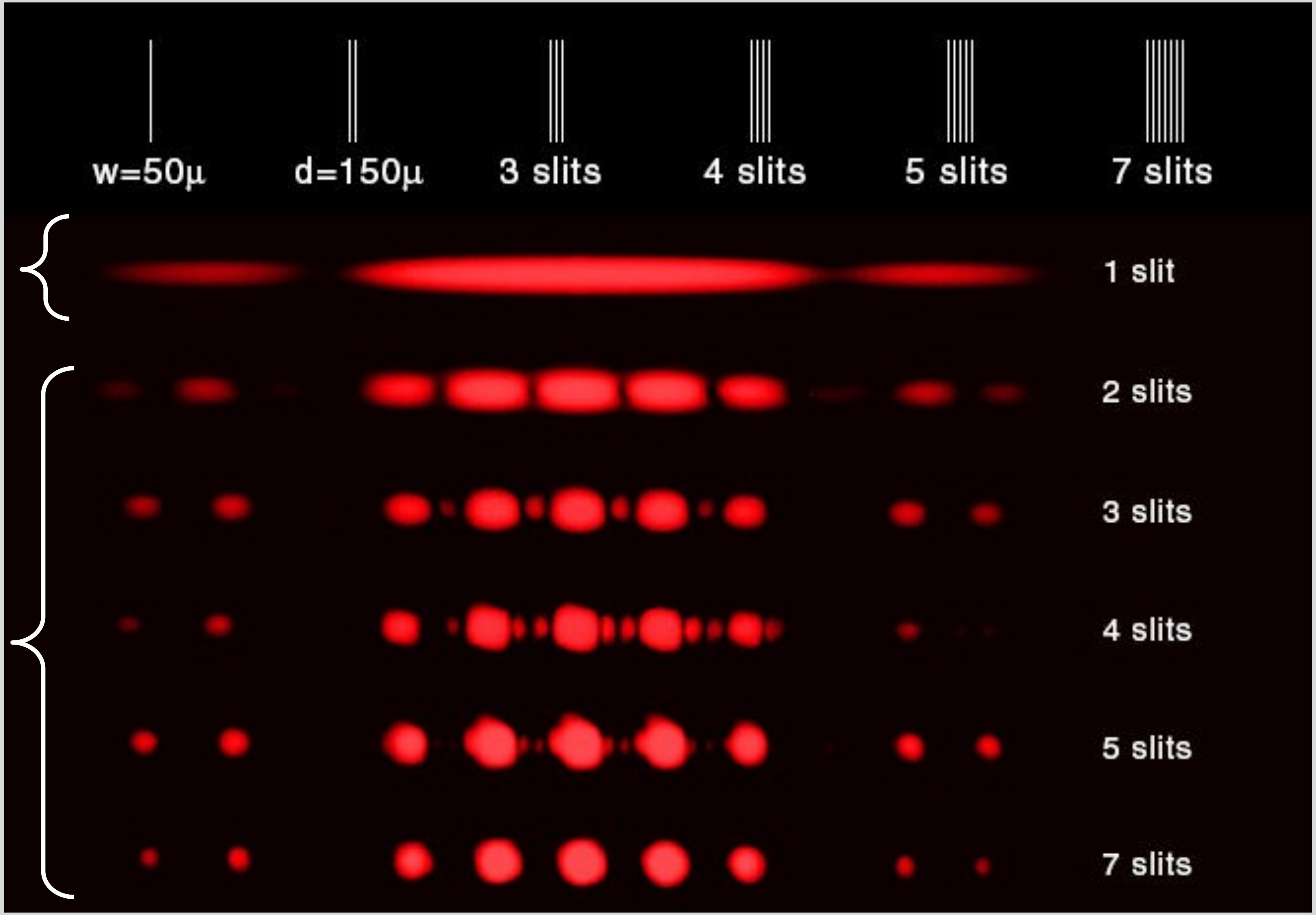
Slit width (a) affects the overall intensity envelope

Multi-slit interference

Keep adding slits:

diffraction
only

diffraction
plus
interference

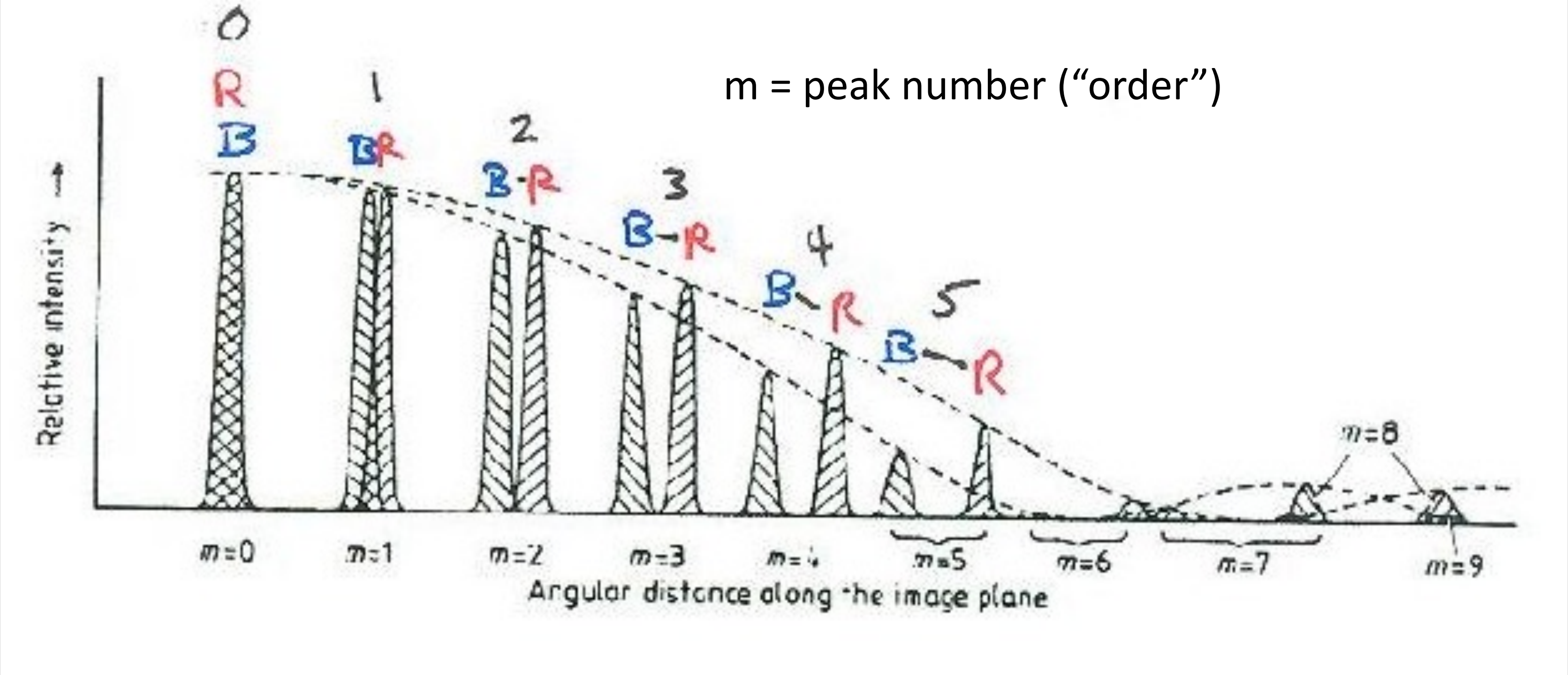


Dispersion in wavelength

So far, we have considered monochromatic light (single fixed λ). Now consider a mythical light source that produces light at two wavelengths only, a blue line and a red line.

Since spacing of peaks is λ/d, blue and red peaks will happen at different places because of the λ-dependence of the interference term.

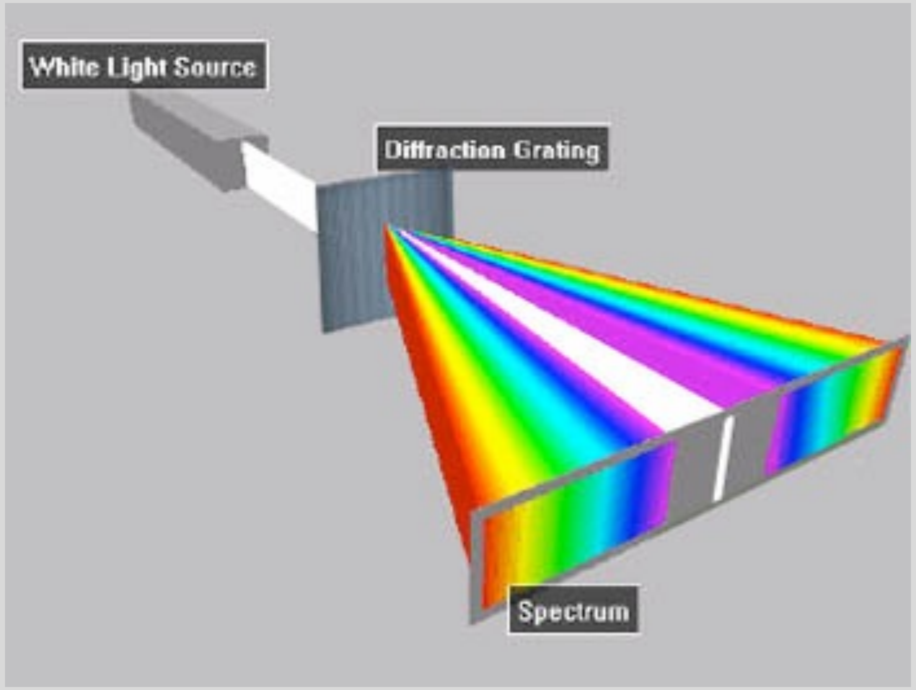
This is referred to as *dispersion*.



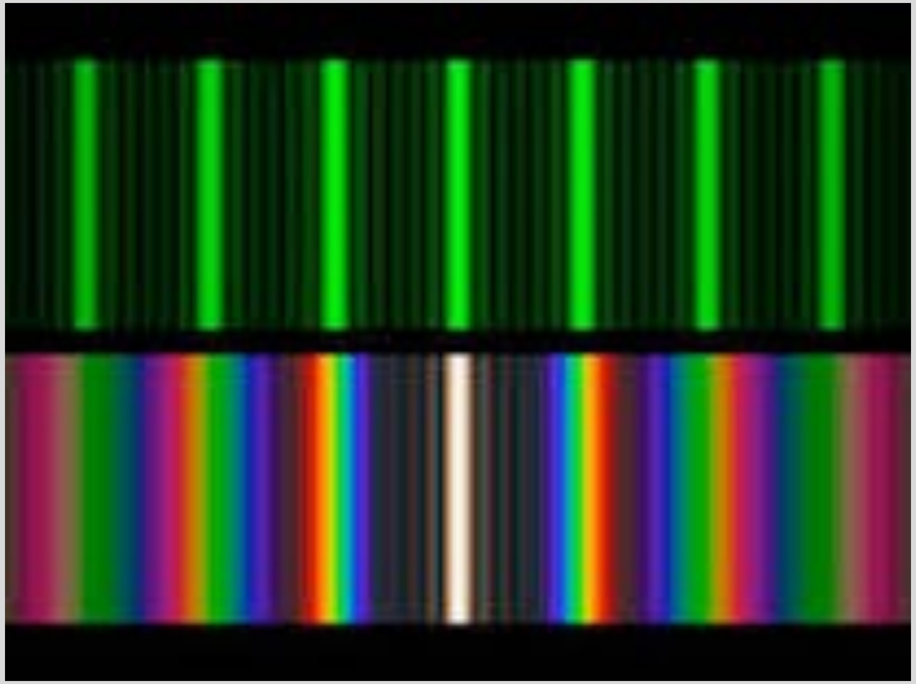
Spectroscopic Instrumentation: Transmission gratings

Now put a continuum (white light) source through a diffraction grating: a transparent medium (film/glass) with fine grooves etched in it.

transmission grating



showing $m = -1, 0, +1$



monochromatic

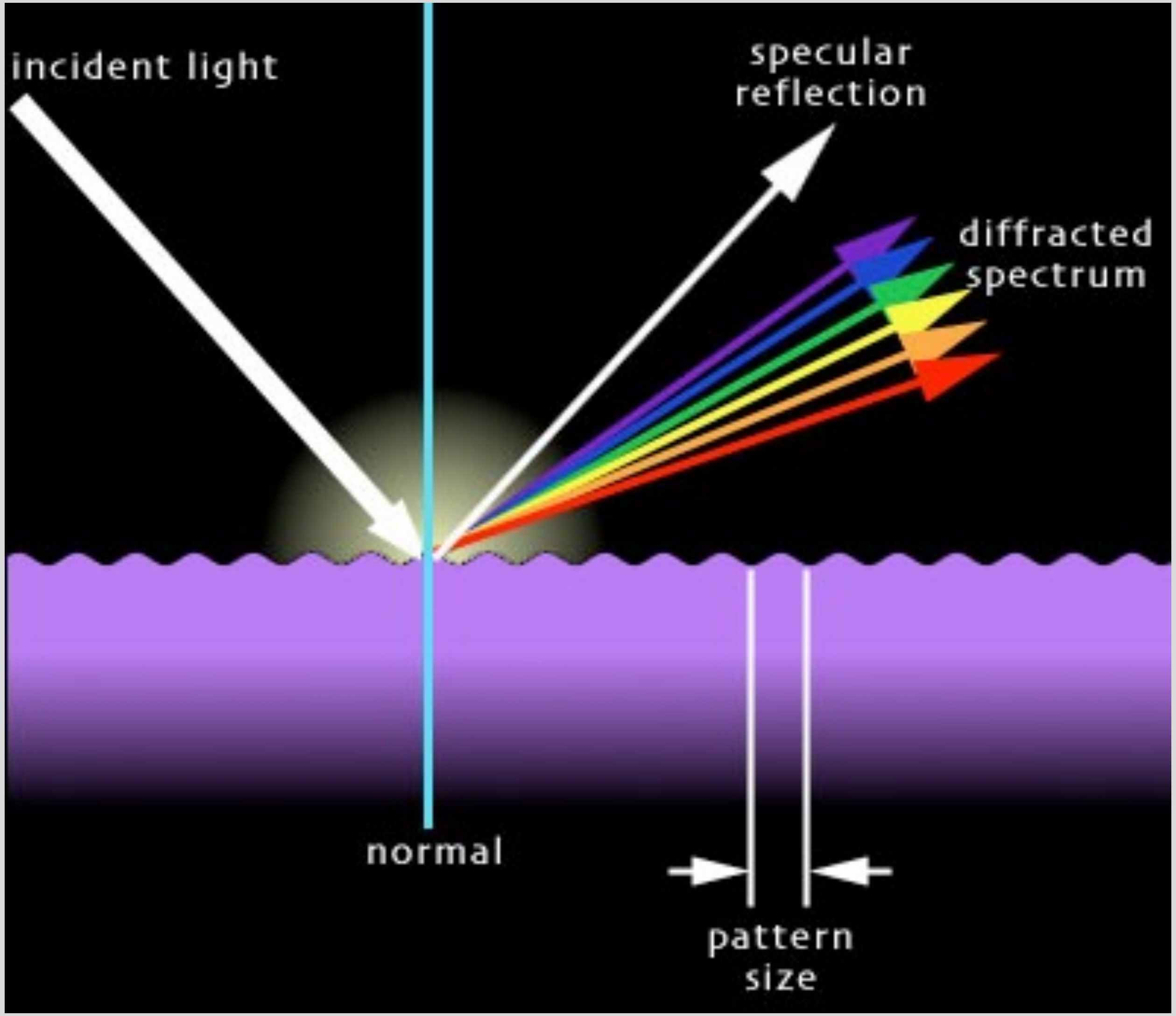
continuum

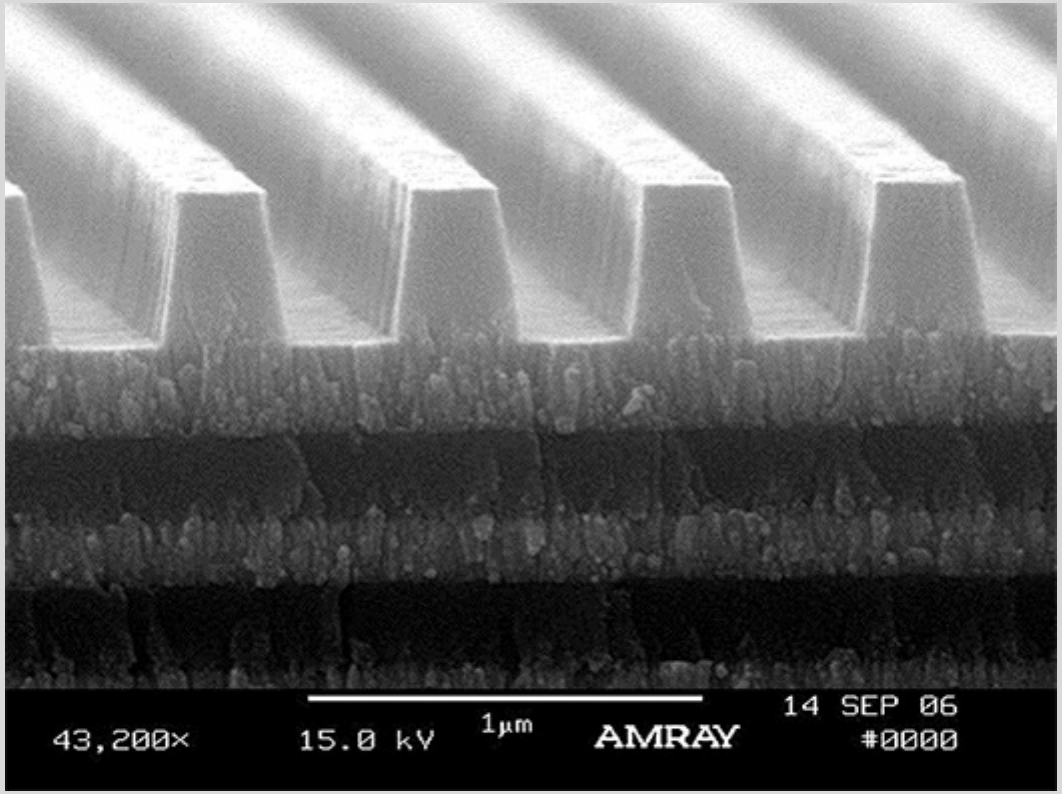
showing $m = -3, -2 - 1, 0, +1, +2, +3$

(Notice increasing dispersion in higher orders!)

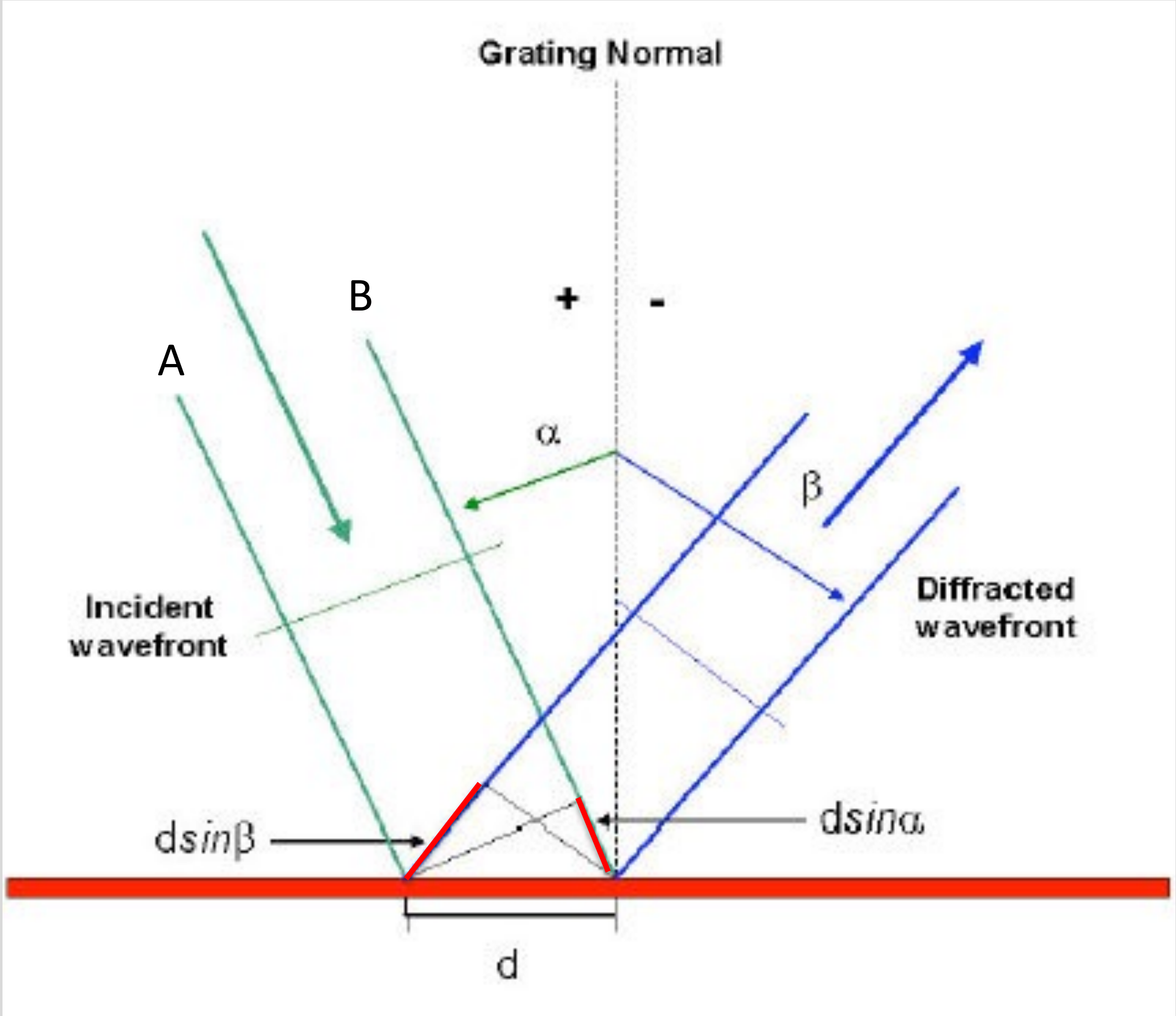
Spectroscopic Instrumentation: Reflection gratings

Alternatively, look at light diffracting off a reflective grooved surface:





Path length differences



α = angle of incidence
β = angle of diffraction
d = groove separation

Consider two rays forming the incoming wavefront, coming in at an angle α and being diffracted out at an angle β.

Ray B has to go an extra distance $d\sin\alpha$ before hitting the grating, and Ray A has to go an extra $d\sin\beta$ after hitting the grating.

Total path length difference is $d\sin\alpha - d\sin\beta$.

But since β is defined to be negative, and sine identities say $\sin(-\beta) = -\sin(\beta)$, we can write this as $d\sin\alpha + d\sin\beta$.

We get **constructive interference** when this path length is an integer multiple of the wavelength:

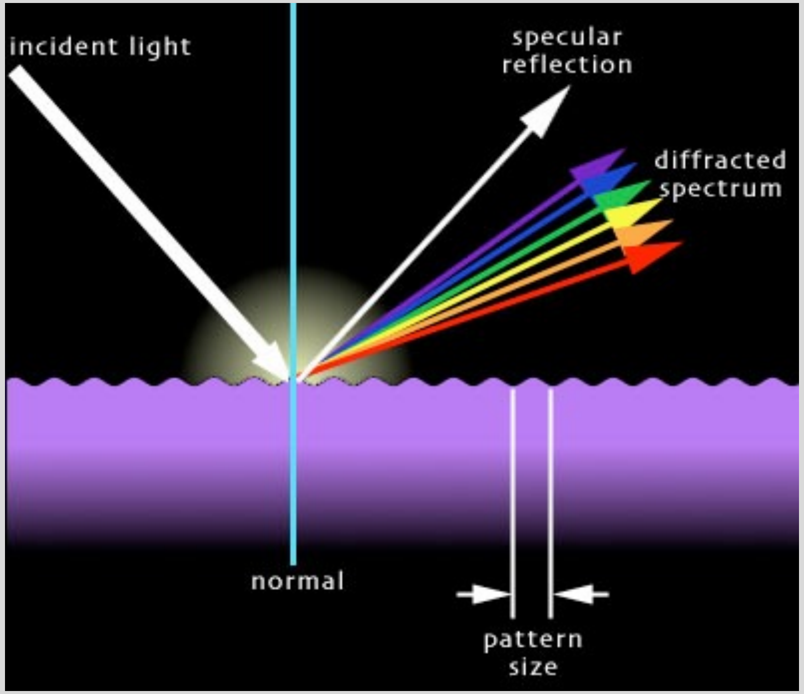
$$\boxed{m\lambda = d\sin\alpha + d\sin\beta}$$

The Grating Equation

$$\boxed{m\lambda = d\sin\alpha + d\sin\beta}$$

for either reflection or transmission gratings!

α = angle of incidence
β = angle of diffraction
d = groove separation
m = order
λ = wavelength



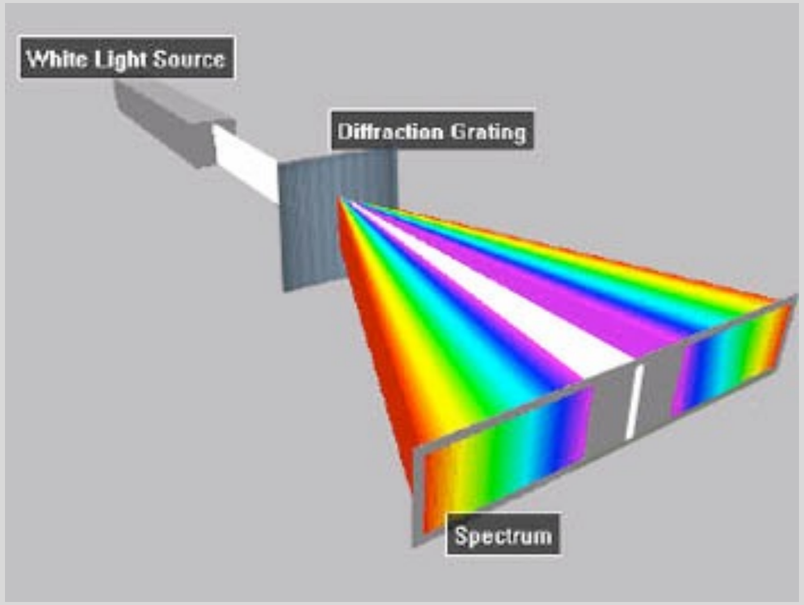
Simplifying checks:

"zeroth order": $m = 0, \;\; \text{so} \;\; \sin(\alpha) = -\sin(\beta)$

no dispersion, just specular reflection or direct transmission

"normal incidence": $\alpha = 0, \;\; \text{so}\; m\lambda = d\sin(\beta)$

dispersion pattern symmetric around m=0



Diffraction grating laser lab

(original from B. Weiner…)

$$m\lambda = d\sin\alpha + d\sin\beta$$

alpha, beta in radians

```
beta=np.arcsin(m*wave/d - np.sin(alpha))
```

alpha, beta in degrees

```
beta=np.degrees(np.arcsin(m*wave/d -
                np.sin(np.radians(alpha))))
```

"Normal" incidence: $\alpha = 0$

$$\beta = \sin^{-1}(m\lambda/d)$$

	Green Laser λ = 5320 Å	Red Laser λ = 6350 Å
Grating: 500 lines/mm, d= 1/500 mm = 20,000 Å		
m = 0	$\beta = 0°$	$\beta = 0°$
m = 1	$\beta = 15.4°$	$\beta = 18.5°$
m = 2	$\beta = 32.1°$	$\beta = 39.4°$
m = 3	$\beta = 52.9°$	$\beta = 72.3°$

Off-axis incidence: $\alpha = 15°$

$$\beta = \sin^{-1}(m\lambda/d - \sin\alpha)$$

	Green Laser λ = 5320 Å	Red Laser λ = 6350 Å
Grating: 500 lines/mm, d= 1/500 mm = 20,000 Å		
m = 0	$\beta = -15°$	$\beta = -15°$
m = +1 m = -1	$\beta = 0.4°$ $\beta = -31.7°$	$\beta = 3.4°$ $\beta = -35.2°$
m = +2 m = -2	$\beta = 15.9°$ $\beta = -52.3°$	$\beta = 22.1°$ $\beta = -63.4°$

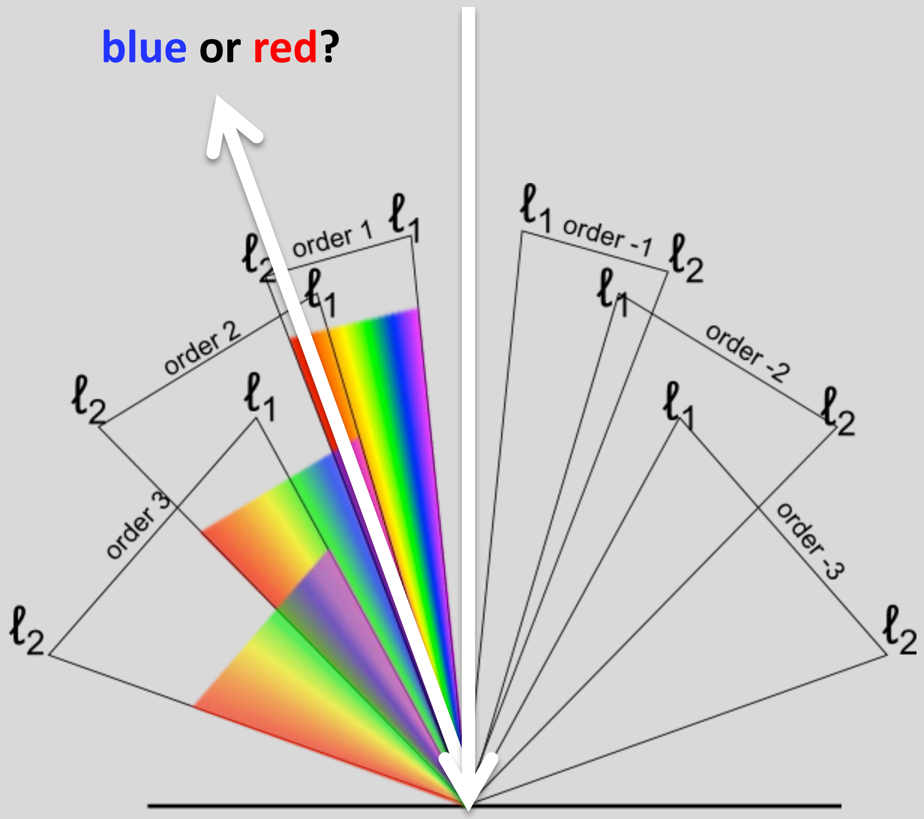
Free Spectral Range

Look at diffracted light in different orders.

For simplicity, let's sketch normal incidence (α=0).

At any given β (outgoing angle), there can be light overlapping from various orders.

For example, in this sketch at $\beta = 20°$ we have both $\lambda_{m=1} = 8000$ Å and $\lambda_{m=2} = 4000$ Å.



Free spectral range: region of spectrum free from overlapping orders

How do we get rid of this problem of overlapping orders?

- Put a ***filter*** in front of the spectrograph to only let certain wavelengths through.
- Put in a ***cross-disperser***

Cross-dispersed Echelle Spectrograph

cross dispersal to separate overlapping orders:

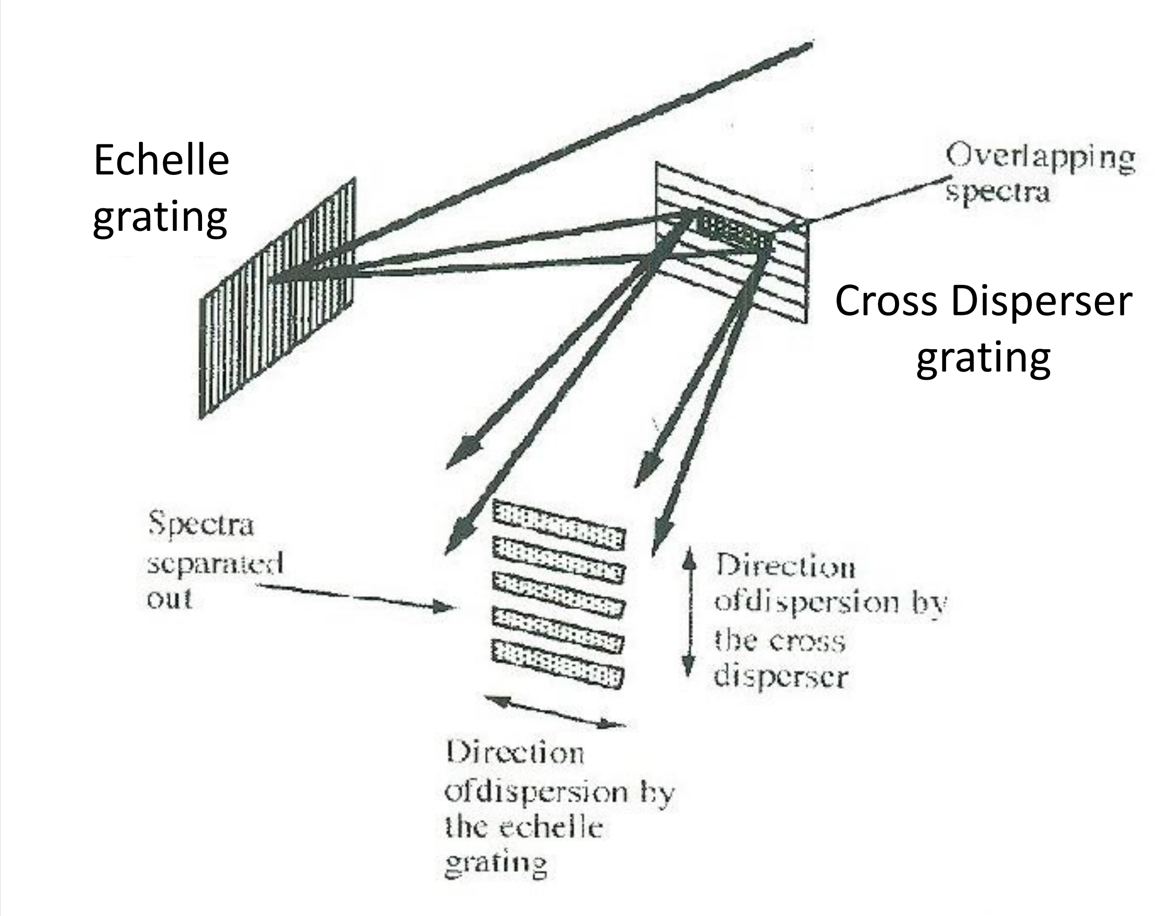
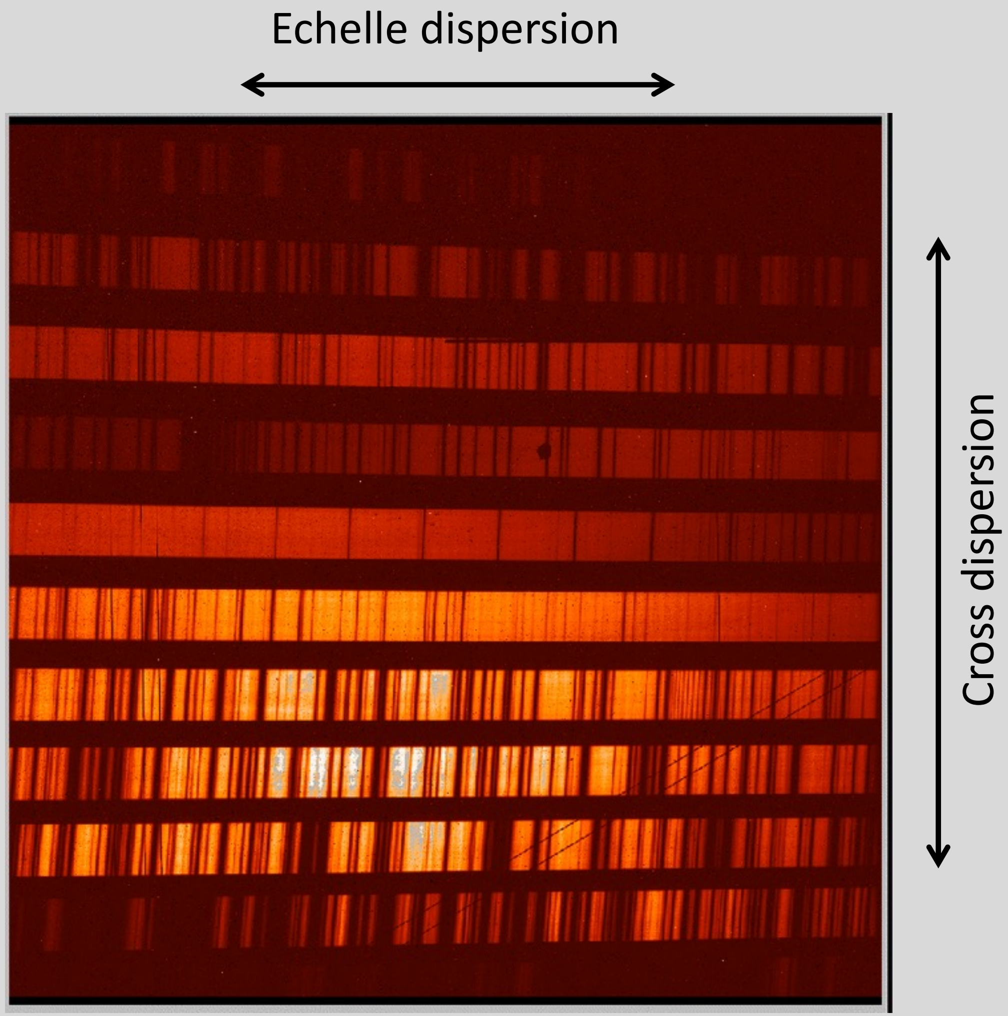


Figure 8.9 Schematic view of an echelle grating and a cross disperser.



Blazing

A grating spreads light out into many orders, and much of which is wasted by not projecting onto the detector.

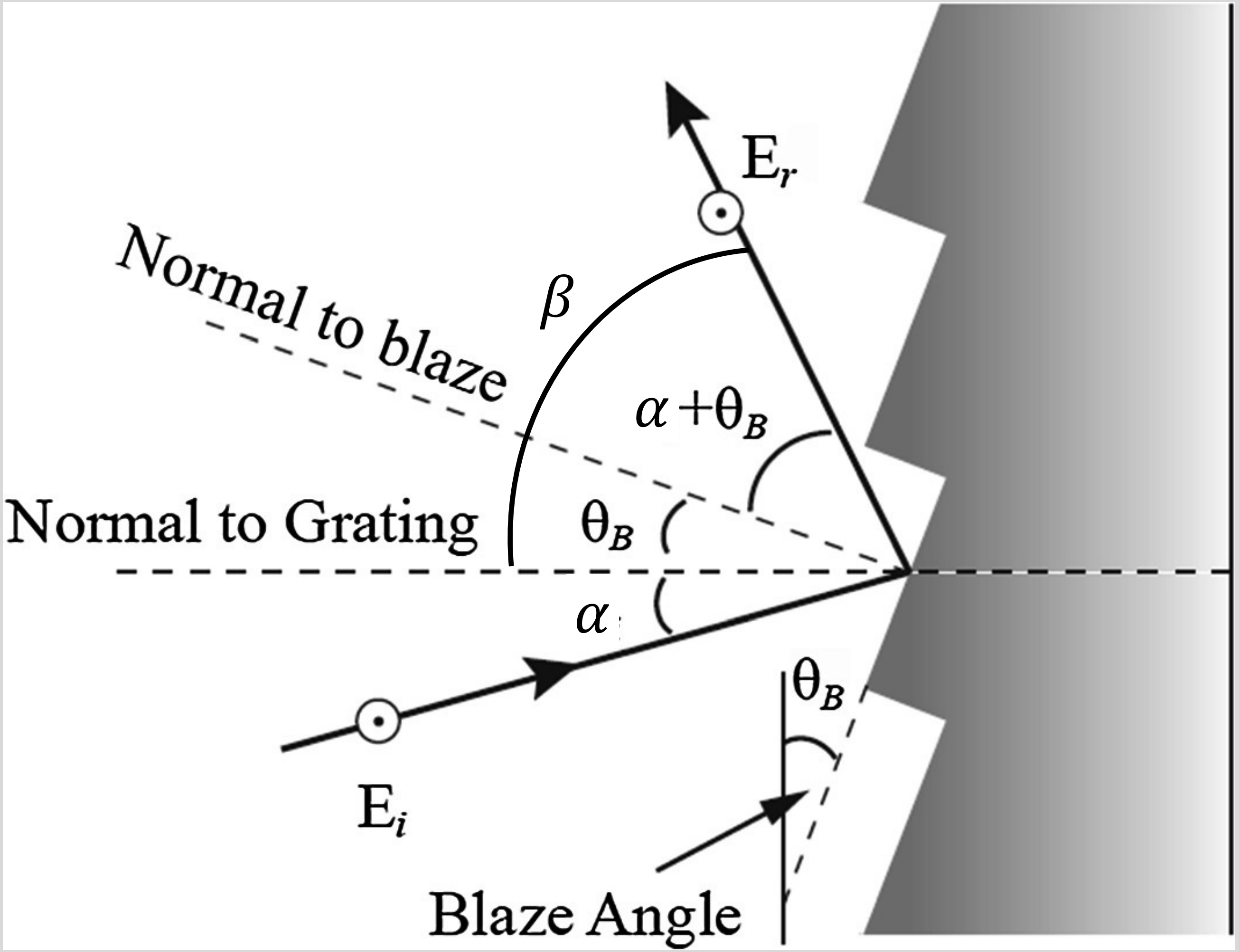
Blazing concentrates ~ 70% light into a particular outgoing angle – a combination of $m\lambda$. Tilt the grooves by an angle θ_B so that the face of the groove points in the direction of the diffraction ray you want to maximize in intensity:

Diffracted light coming out at an angle $\beta = \alpha + 2\theta_B$ will be maximized in brightness. So the "blaze peak" happens when $\alpha + \beta = 2\theta_B$.

(remember that α and β are defined with opposite signs, so in doing the math above, $\alpha - \beta$ becomes $\alpha + \beta$)

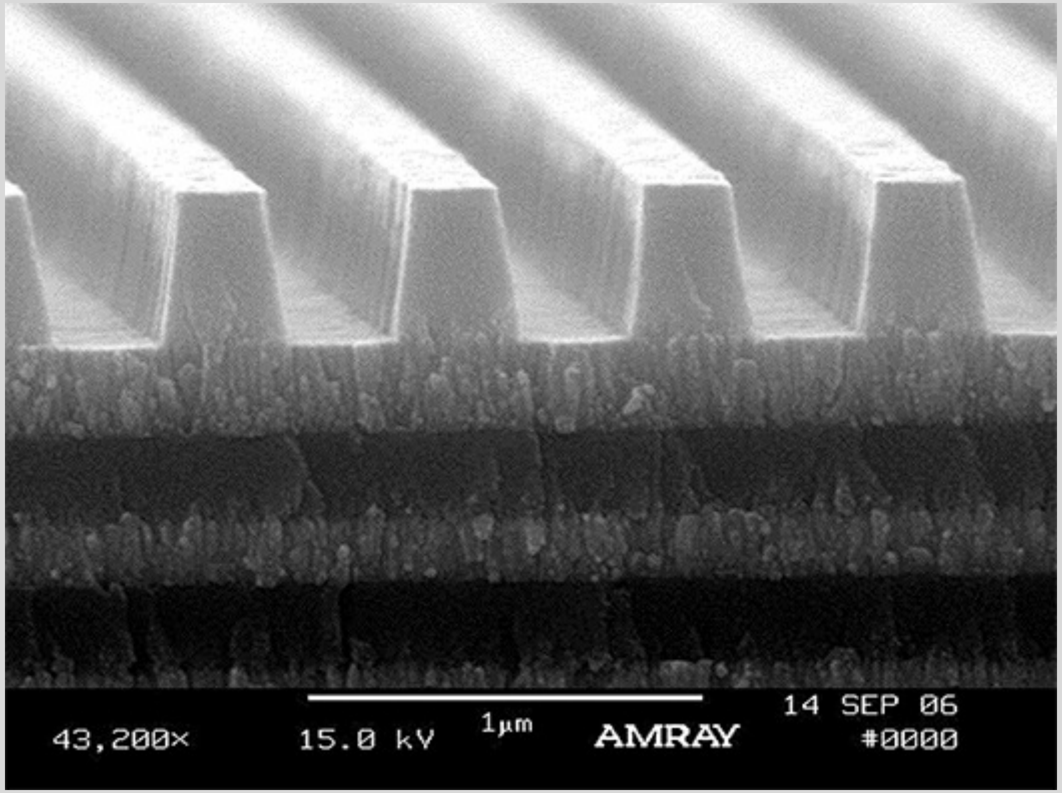
The wavelength that corresponds to that peak brightness is referred to as the blaze wavelength (λ_b) and can be solved using the grating equation to get:

$$m\lambda_b = 2d\cos(\alpha - \theta_B)$$

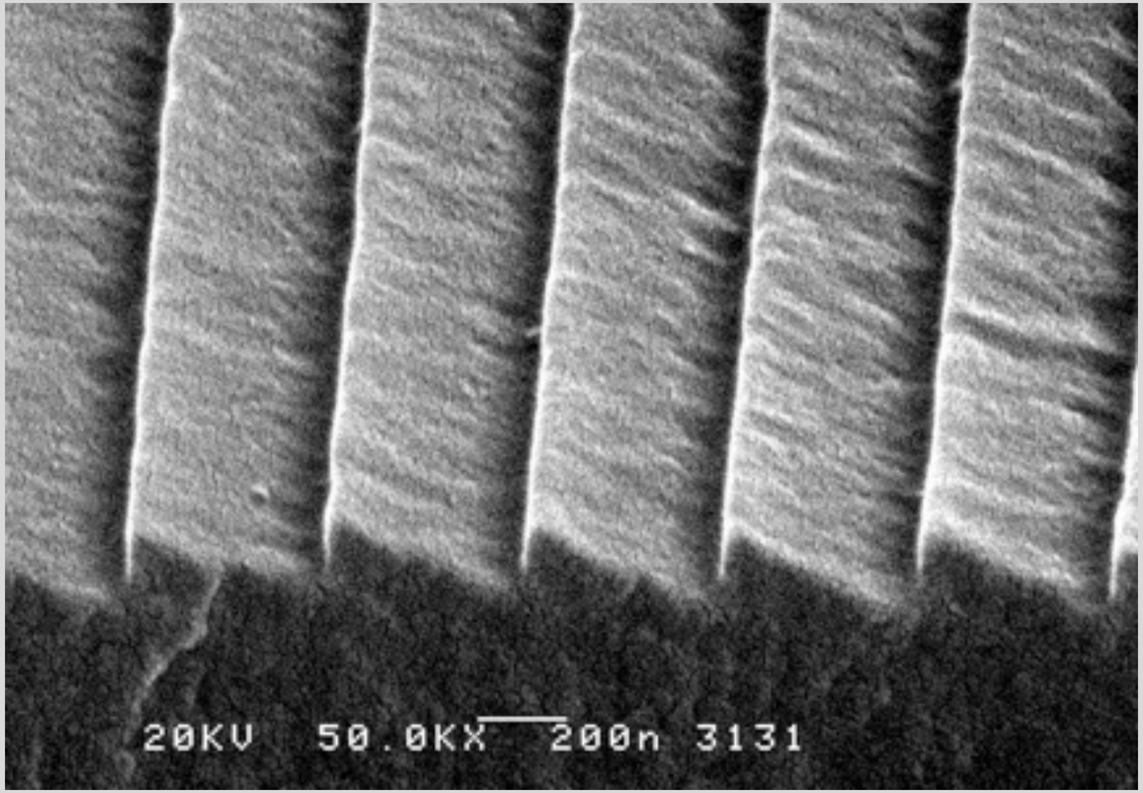


Grating Surfaces

regular (unblazed) grating

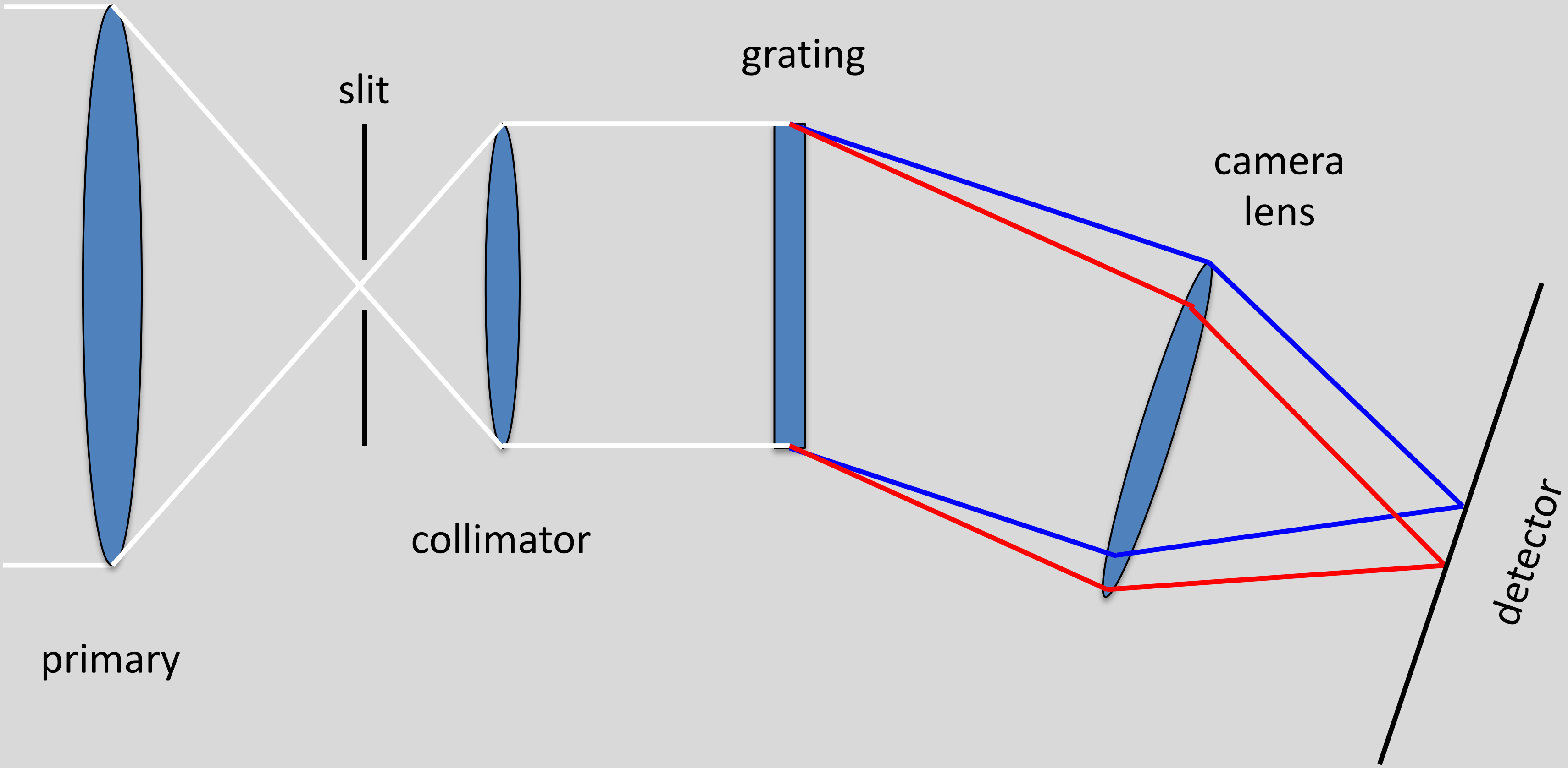


blazed grating



$$m\lambda = d \sin \alpha + d \sin \beta$$

Simple Grating Spectrograph

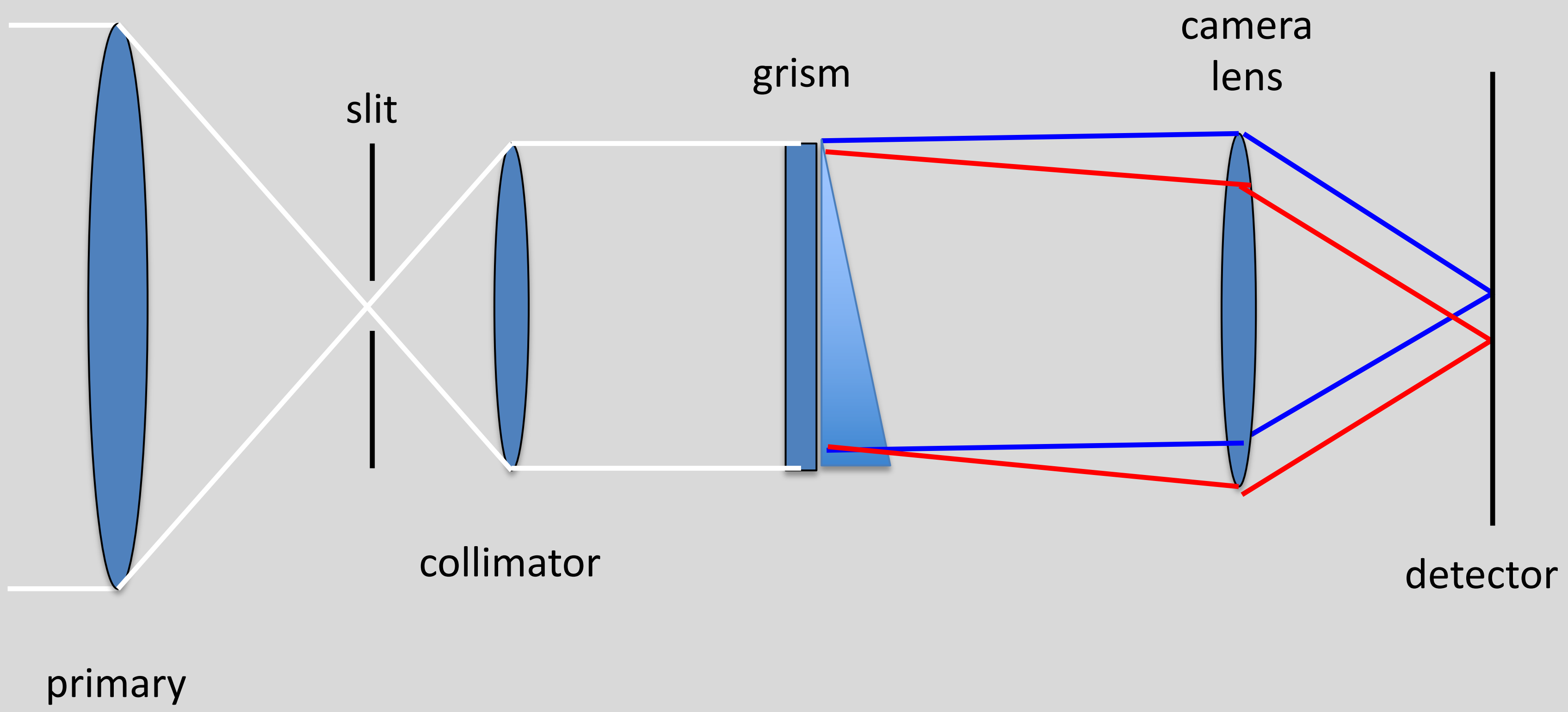


Notes:

1) For ease of sketching, this shows a transmissive system (refracting telescope, transmission grating). Most telescopes use a reflecting system.

2) the focal ratio of primary and collimator must be matched!

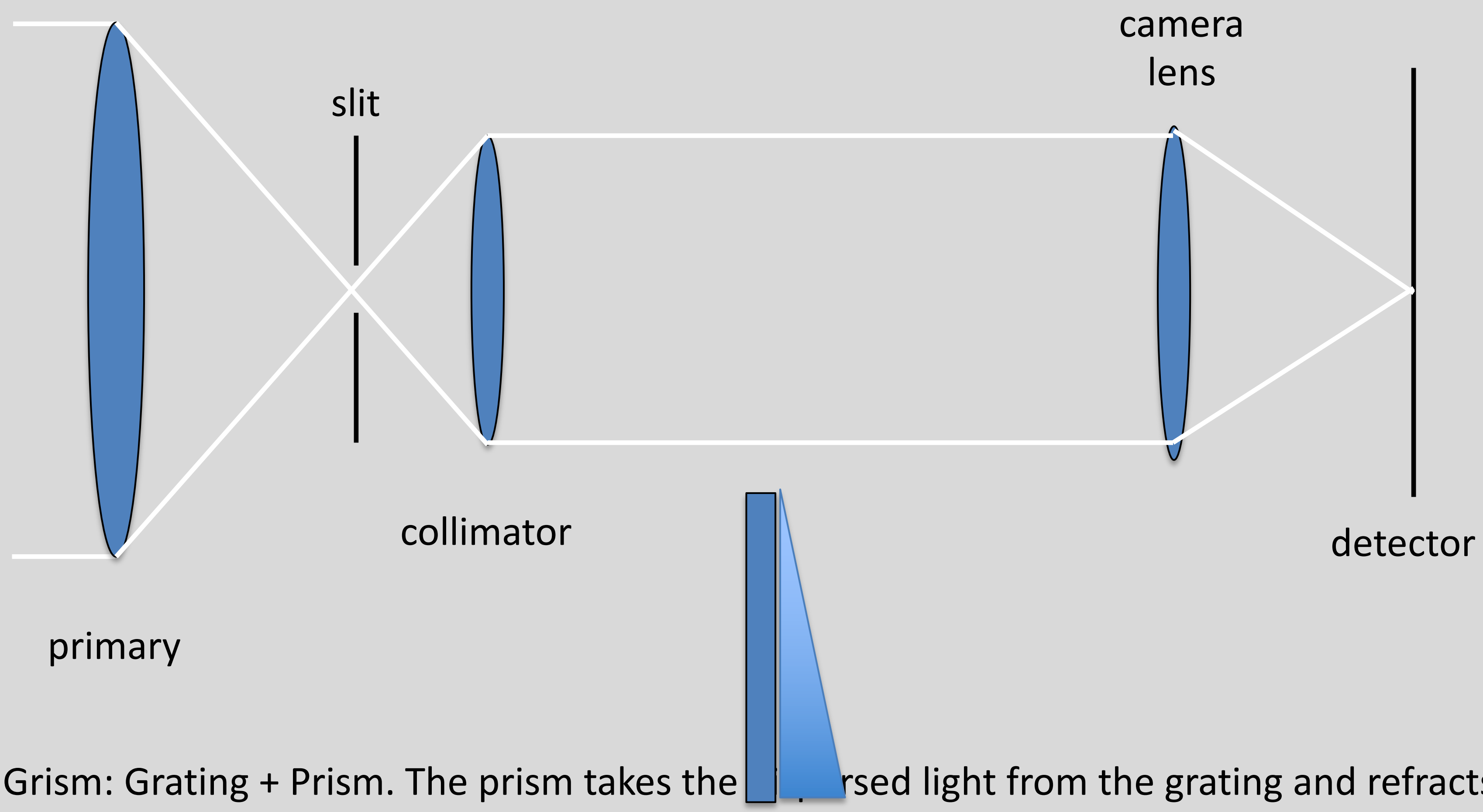
Simple Grism Spectrograph



Grism: Grating + Prism. The prism takes the dispersed light from the grating and refracts it back to a straight-line path.

You can then slide the grism in and out to switch seamlessly between spectroscopy and imaging.

Simple Grism Spectrograph



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