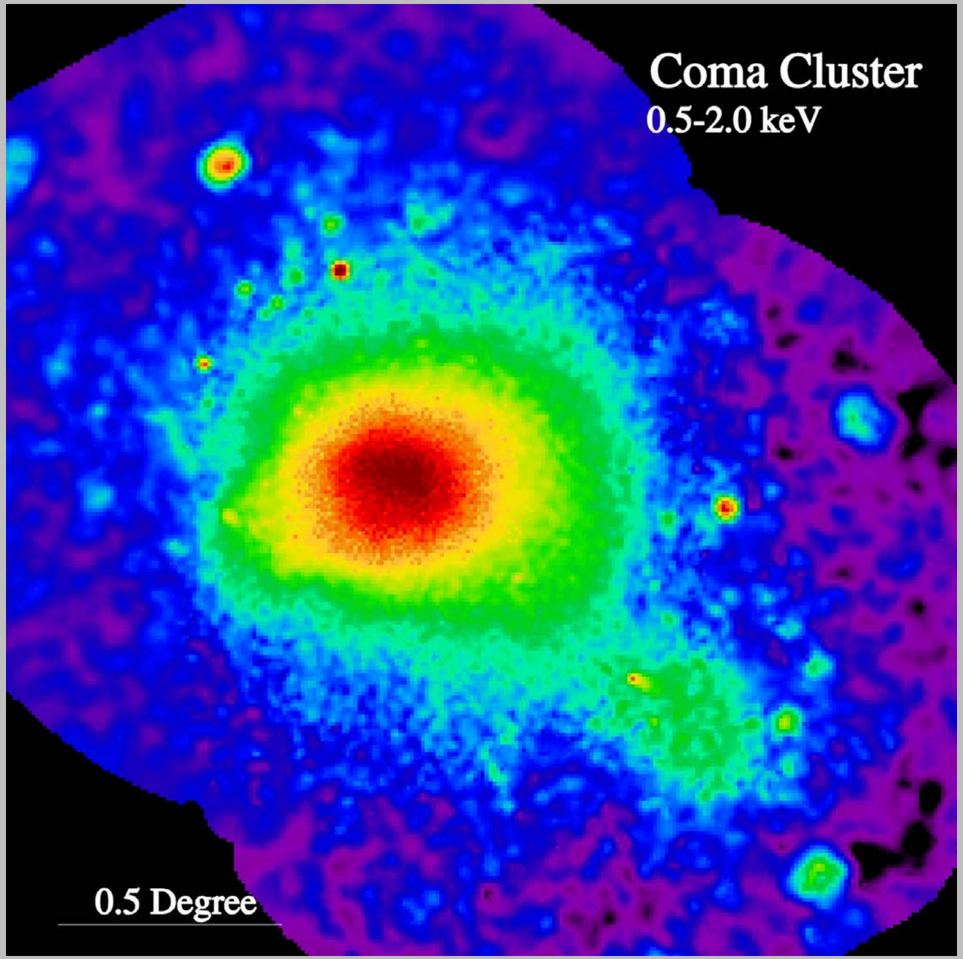
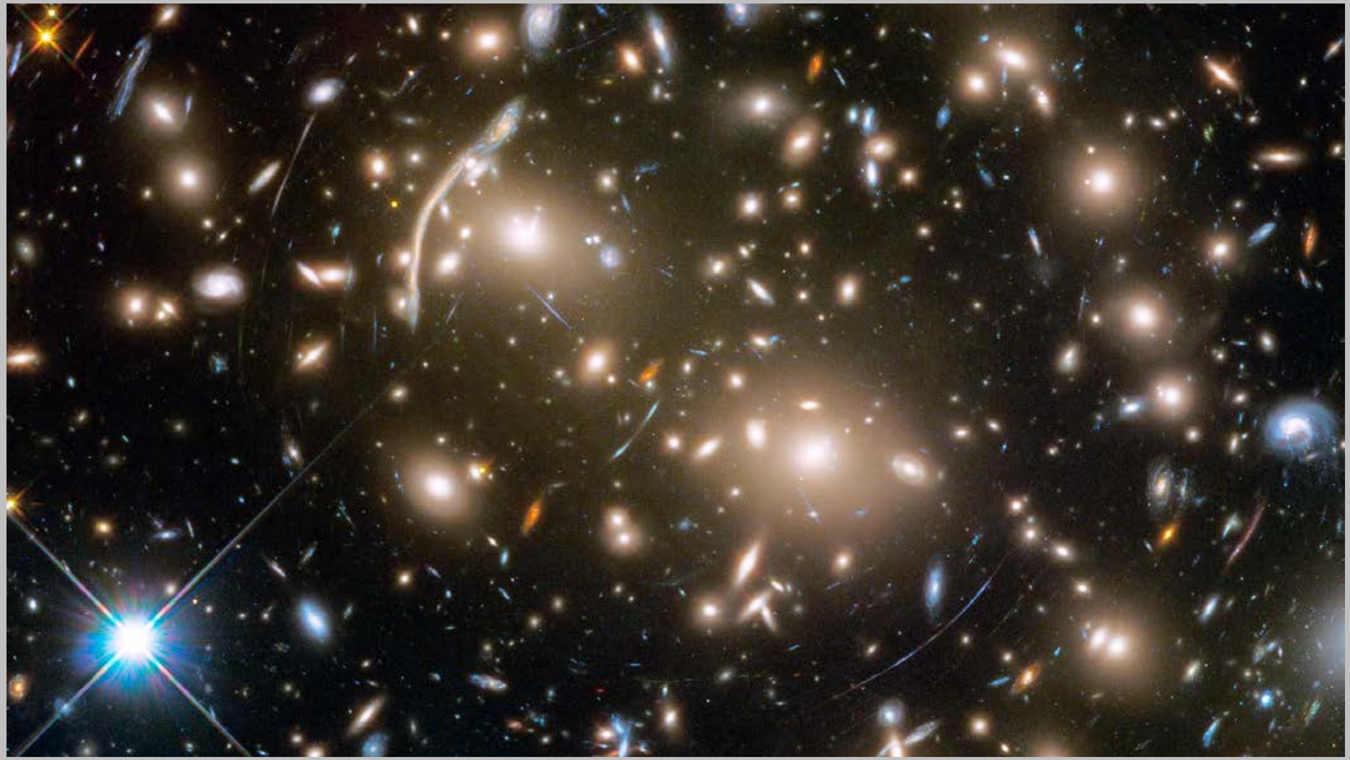
Galaxy Cluster Masses: Other (more recent) methods

Hydrostatic equilibrium: Balance thermal energy of hot X-ray gas with gravitational potential energy of cluster.



Gravitational lensing: the mass of the cluster bends the light from background galaxies, distorting their shapes. This can be modeled to get the cluster mass.



Galaxy cluster mass balance (rough numbers):

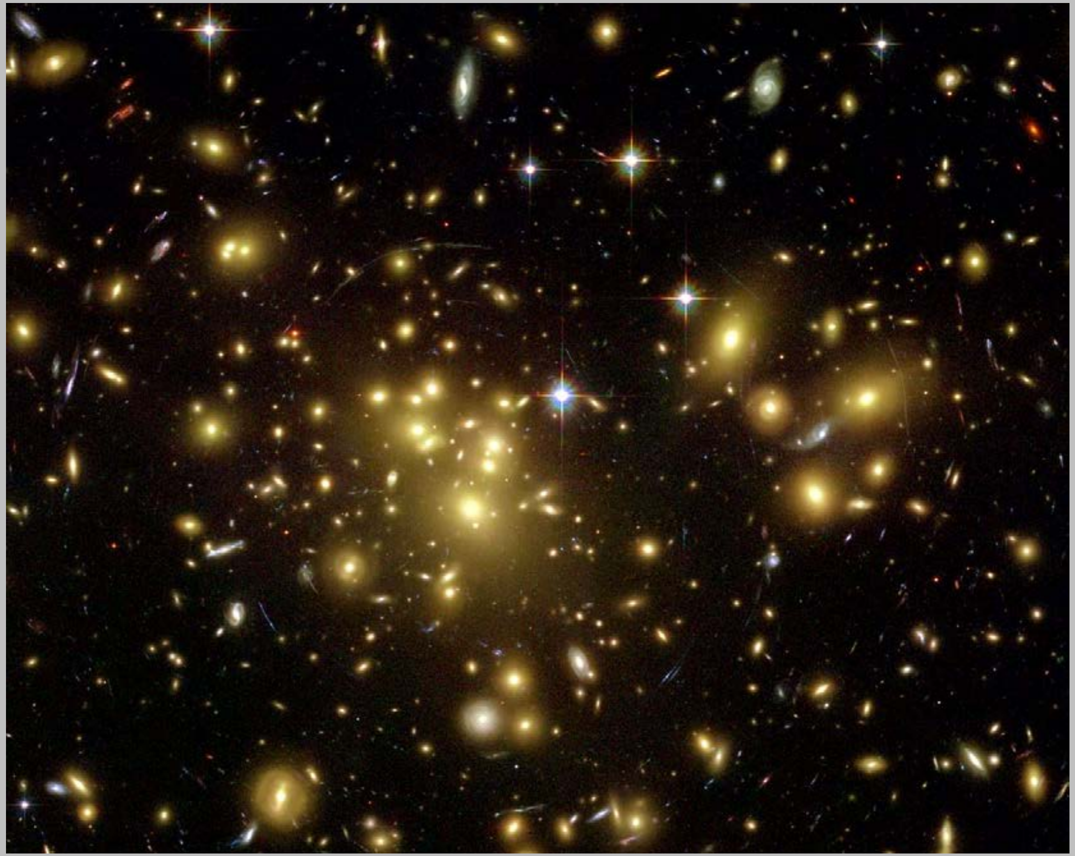
- ≈ 10% of total mass is in stars
- ≈ 20% of total mass is in hot gas
- ≈ 70% of total mass is ”missing”: dark matter

Galaxies: Morphology-Density Relationship

In the local universe, the fraction of galaxy types is a strong function of local environment.

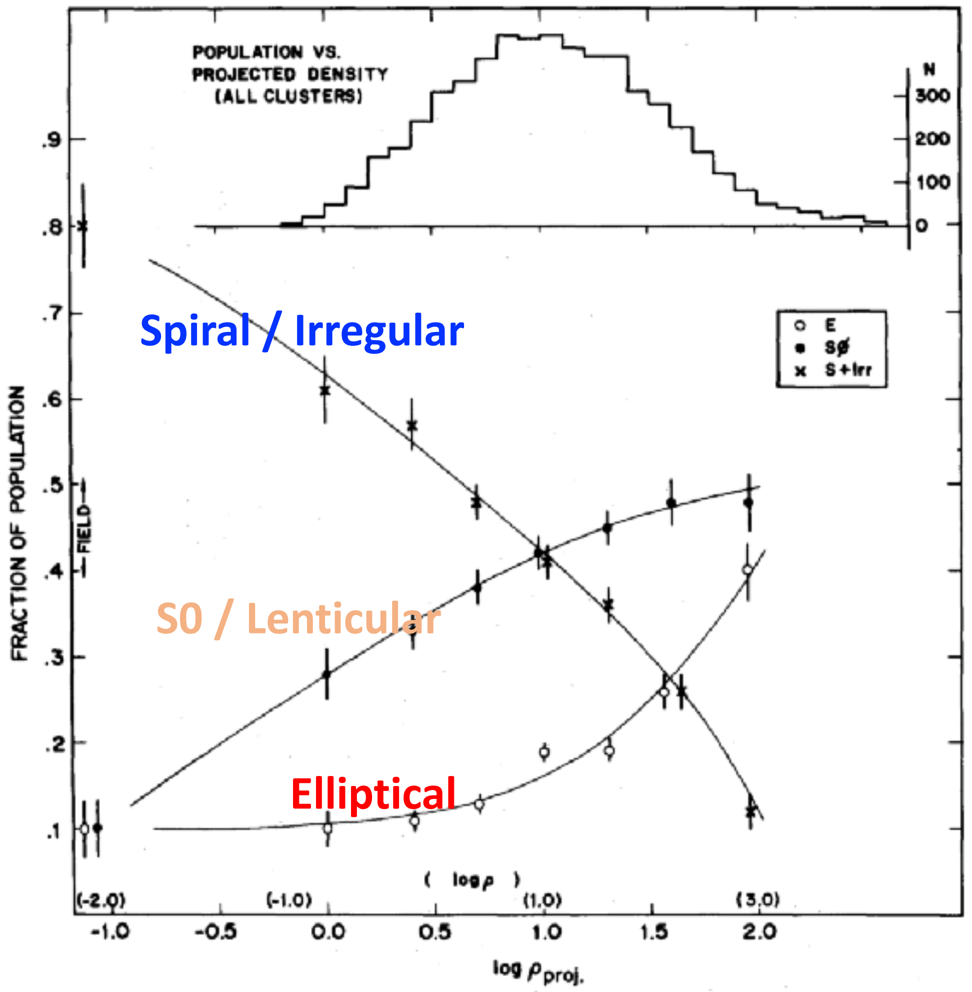
Spirals/Irregulars dominate the in the field environment.

S0's and E's dominate in galaxy clusters.



Dressler 1980

Galaxy Population Fraction



Projected Number Density of Galaxies
log(# per Mpc^2)

Evolution of Cluster Galaxies

Hubble Space Telescope (and now JWST) lets us look at galaxies in distant clusters, to see how things have changed with time.

Higher fraction of star-forming spiral galaxies in the past and a higher fraction of “red and dead” E and S0 galaxies today: galaxy evolution!

HST image of galaxy cluster MACS J0717+3745. Redshift $z = 0.55$, so we are looking back in time ≈ 5 billion years.

Cluster Galaxy Evolutionary Processes

1) **Collisions and mergers**: galaxies interact, collide, and sometimes even merge in group and cluster environments.

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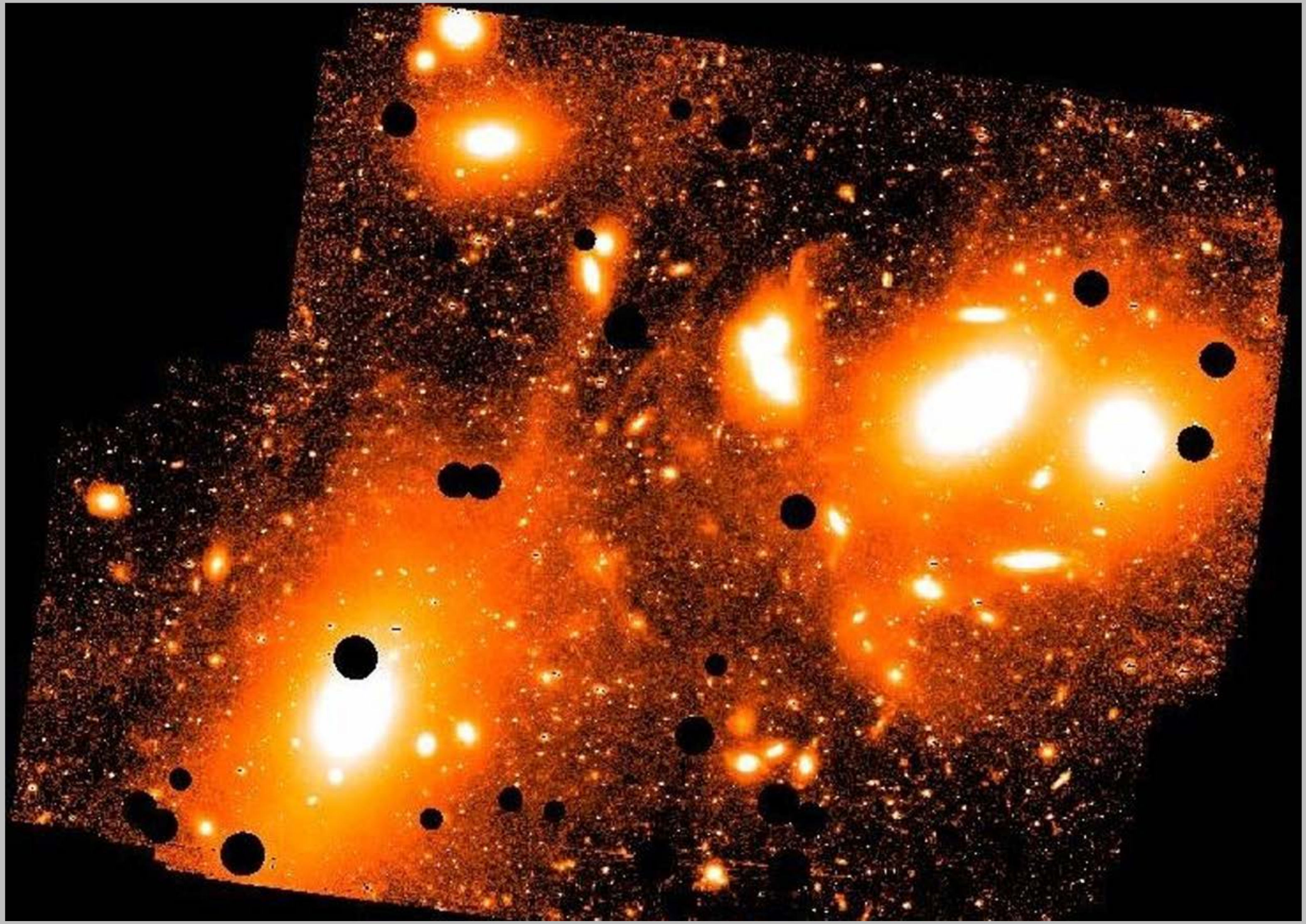
2) **Tidal stripping**: the tidal forces from the cluster's gravitational potential as a whole strips stars from galaxies and even completely shred smaller galaxies.



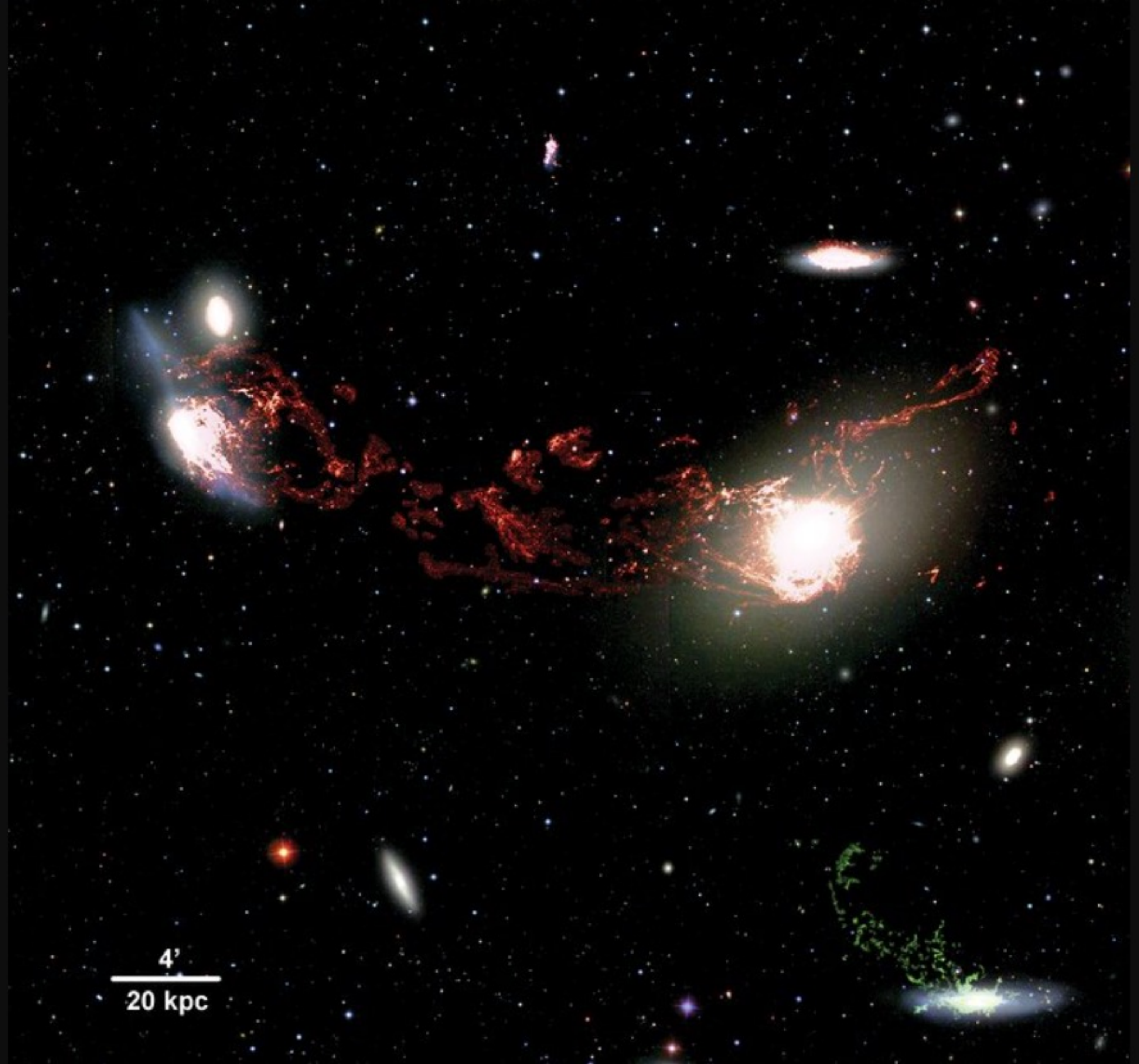
Tidal streams in the Virgo Cluster (Mihos+ 05)

Cluster Galaxy Evolutionary Processes

1) **Collisions and mergers**: galaxies interact, collide, and sometimes even merge in group and cluster environments.

2) **Tidal stripping**: the tidal forces from the cluster's gravitational potential as a whole strips stars from galaxies and even completely shred smaller galaxies.

3) **Ram pressure stripping**: In massive clusters, the gas pressure of the hot X-ray gas can completely strip cold star-forming gas out of spiral galaxies as they move through the cluster.



Red and green: filaments of gas stripped out of spiral galaxies in Virgo

[Kenney+ 08]

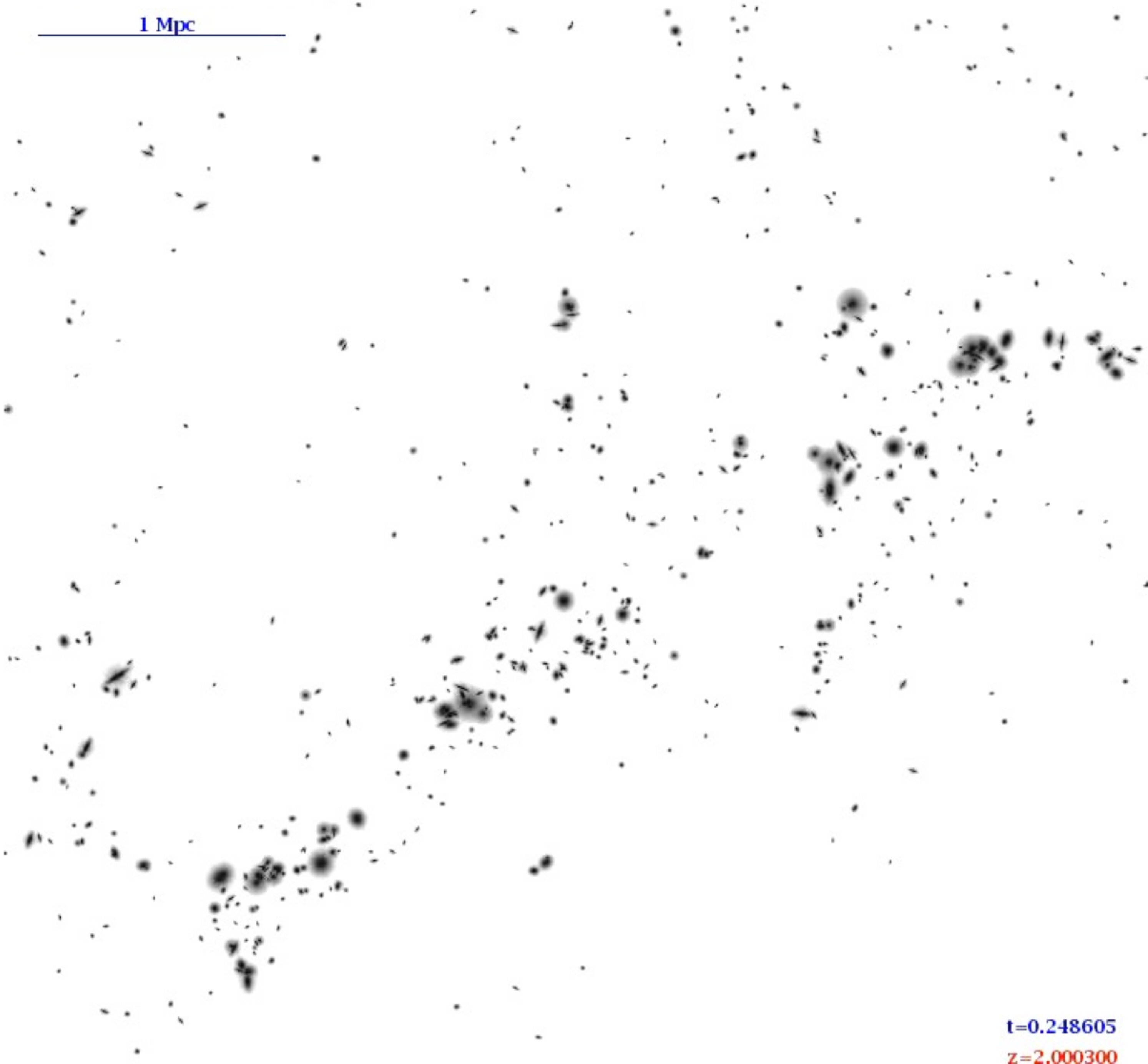
Galaxy Cluster Formation

Clusters grow over time as gravity pulls galaxies together.

Hierarchical accretion: small groups form first, then groups of galaxies merge to form small clusters, then small clusters merge together to form big clusters.

Cluster formation is an *on-going process*.

Rudick+ 11



Formation of a galaxy cluster over ≈ 10 billion years of time

Large Scale Structure

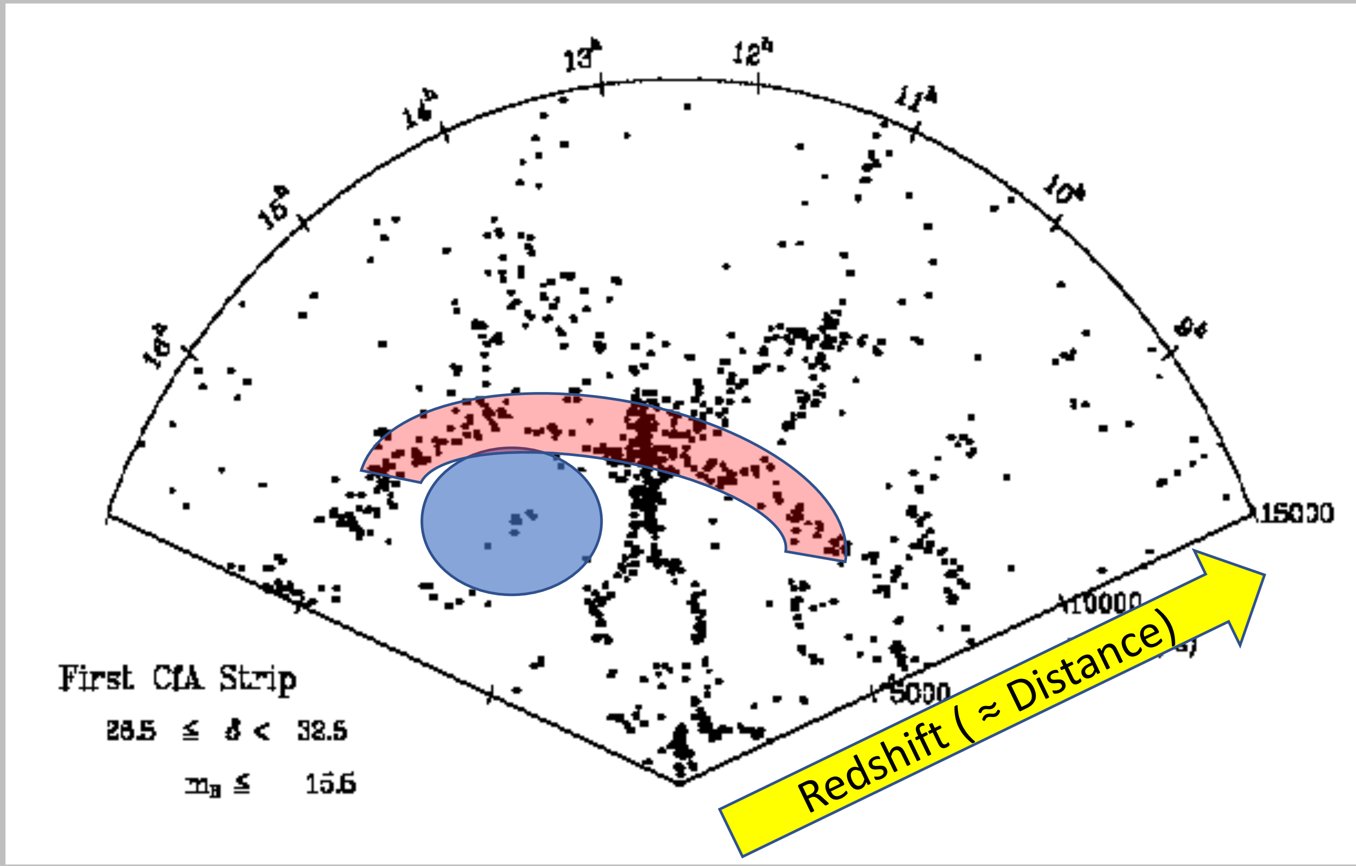
On scale of 100s of Mpc, galaxies are not spread completely randomly through space: they come in clusters, or strung out along “filaments”, along with “voids”: regions of very low galaxy density.

1985: The Stickman Cometh

The Harvard/Smithsonian Center for Astrophysics produces one of the first redshift surveys, mapping out redshifts for a large sample of galaxies to look at their spatial distribution ⇒

The Bootes Void: 100 Mpc away, 75 Mpc in diameter.
The Great Wall: 200 Mpc across, 10 Mpc thick

The CfA redshift survey: a polar plot of the galaxy distribution: redshift (distance) on the radial coordinate and angle across the sky on the angular coordinate.



Large Scale Structure

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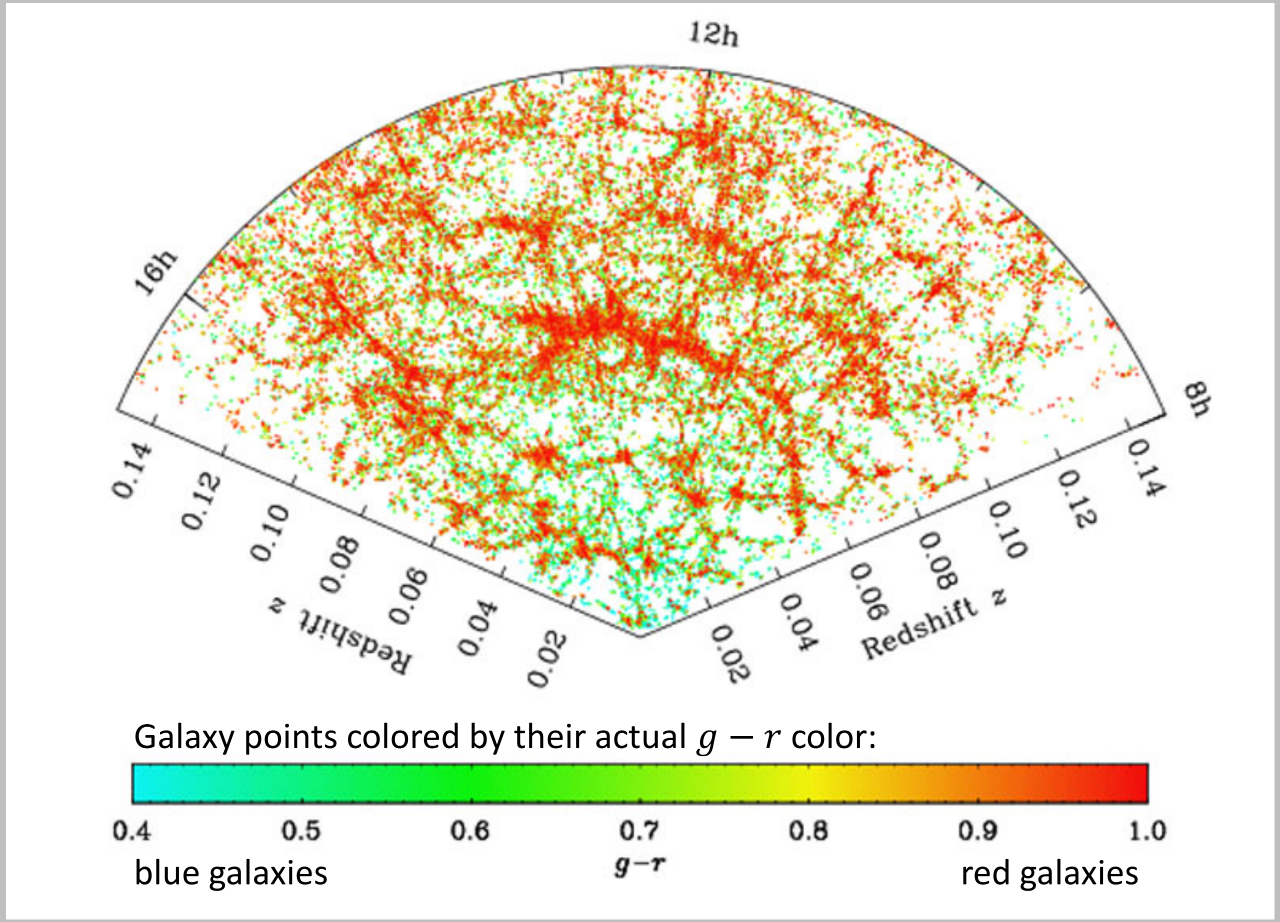
2000s: Sloan Digital Sky Survey

Many more galaxies, much deeper survey going to much larger distances.

The galaxy distribution shows lots of structure: think of cutting though a sponge.

Red galaxies more highly clustered than blue galaxies.

SDSS redshift survey: like the CfA survey, but much deeper and with many more galaxies. (Zehavi+ 11)



What causes all this structure? 13.6 billion years of gravity acting on cosmic scales.

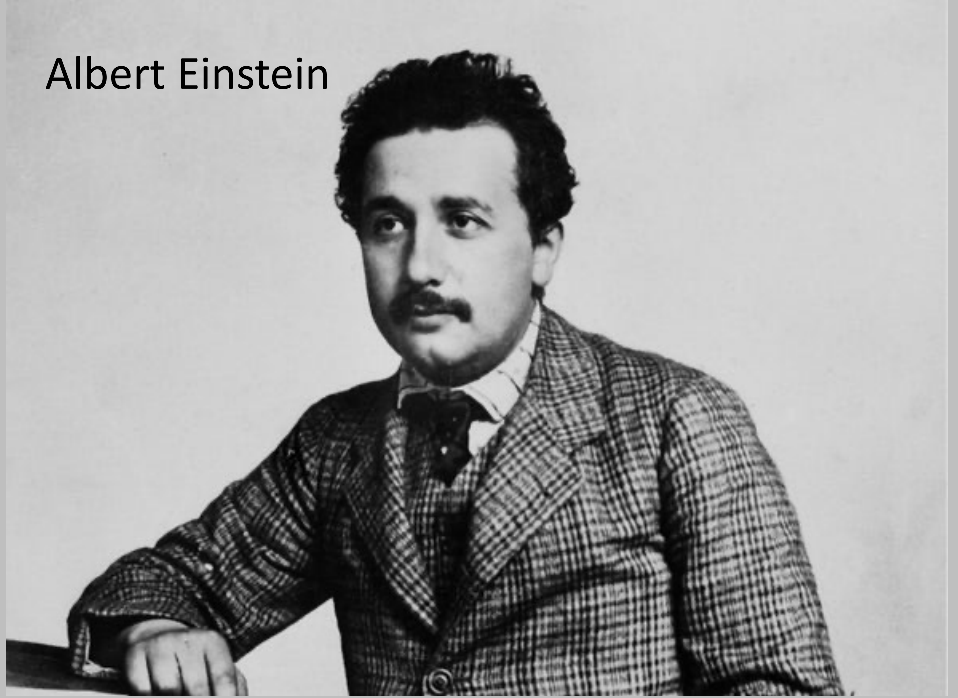
But to understand this.... *it's time for cosmology.*

Cosmology

The beginnings of modern cosmology

1915: Einstein works out the model of General Relativity, describing gravitation and its coupling with space and time.

Willem de Sitter uses GR to work out a model that has the Universe expanding in time. Einstein rejects that concept, adding an ad-hoc term to his equations called the cosmological constant which keeps the universe from expanding.



The beginnings of modern cosmology

1915: **Einstein** works out the model of General Relativity, describing gravitation and its coupling with space and time.

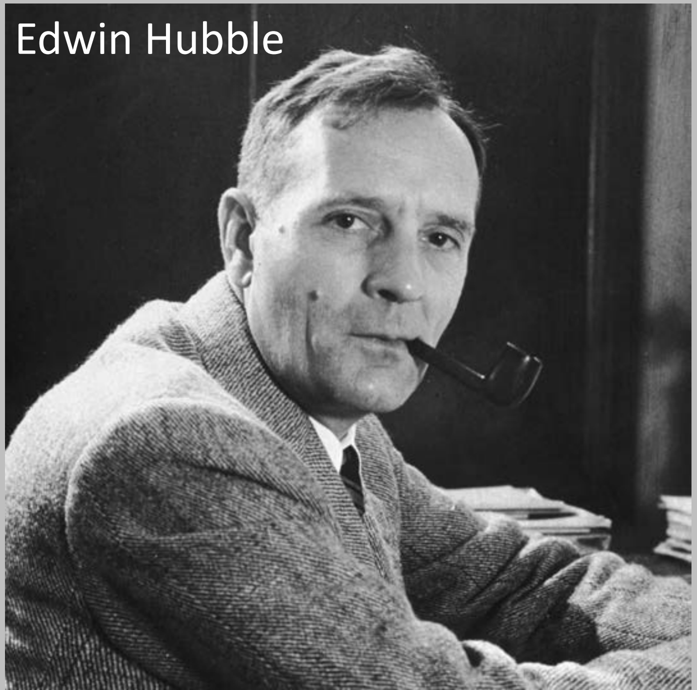
Willem **de Sitter** uses GR to work out a model that has the Universe expanding in time. Einstein rejects that concept, adding an ad-hoc term to his equations called the cosmological constant which keeps the universe from expanding.

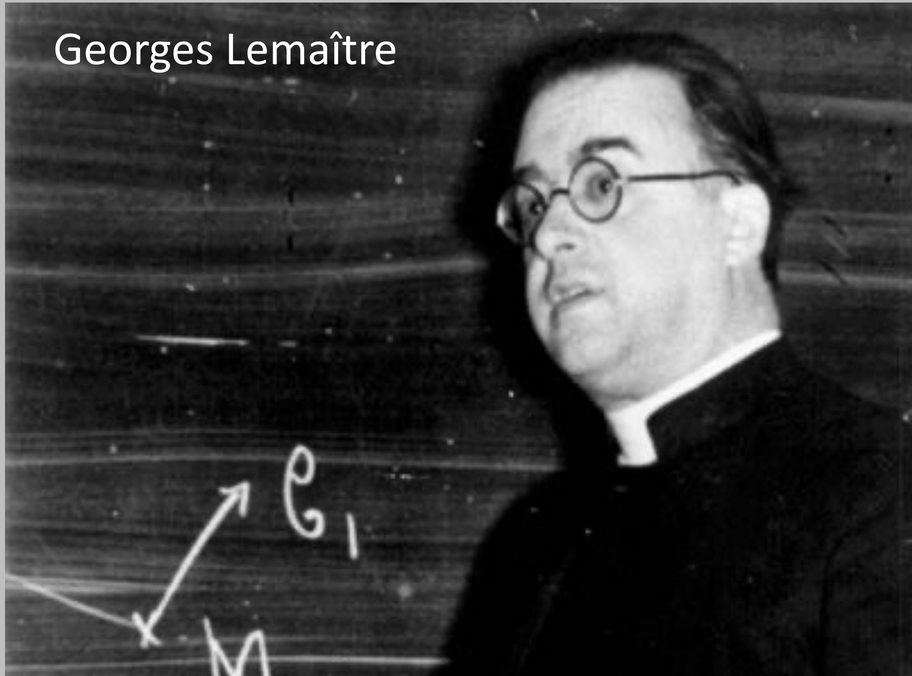
1920s: Alexander **Friedman** and Georges **Lemaître** also put forth models of an expanding universe.

1929: Edwin **Hubble** discovers the expansion of the Universe.

Einstein gives up on the cosmological constant, calling it his "greatest blunder."


Alexander Friedmann

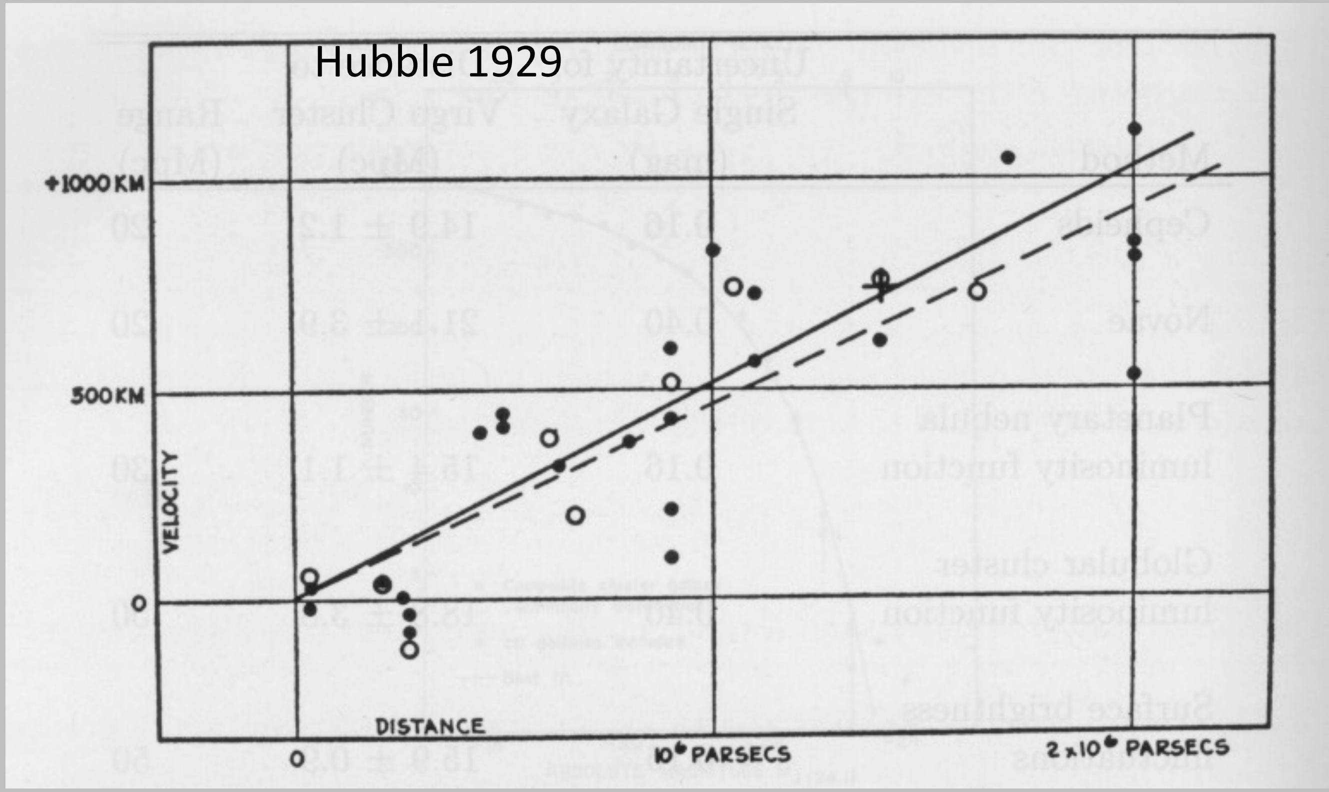

Edwin Hubble


Georges Lemaître

The Expanding Universe

Hubble's Law: $v = H_0 d$, where $H_0 \approx 72$ km/s/Mpc

(Note: Hubble got H_0 wrong, by a lot. H_0 is the slope of that line ⇒ which is about 500 km/s/Mpc. Hubble's incorrect distances were the problem.....)



The true meaning of redshift: cosmological expansion

Remember redshift:

$$z = \frac{\lambda_{obs} - \lambda_0}{\lambda_0} = \frac{\Delta\lambda}{\lambda_0}$$

We originally referred to this using the Doppler shift, but ***this is not correct***. The Universe – space itself – is expanding. As light moves through an expanding Universe, its wavelength is stretched: a redshift.

Important Note: on small scales the expansion is weak and gravitational forces can overcome it. The solar system is not expanding, galaxies are not growing bigger, even galaxy clusters do not expand.

You are not (cosmologically) expanding!

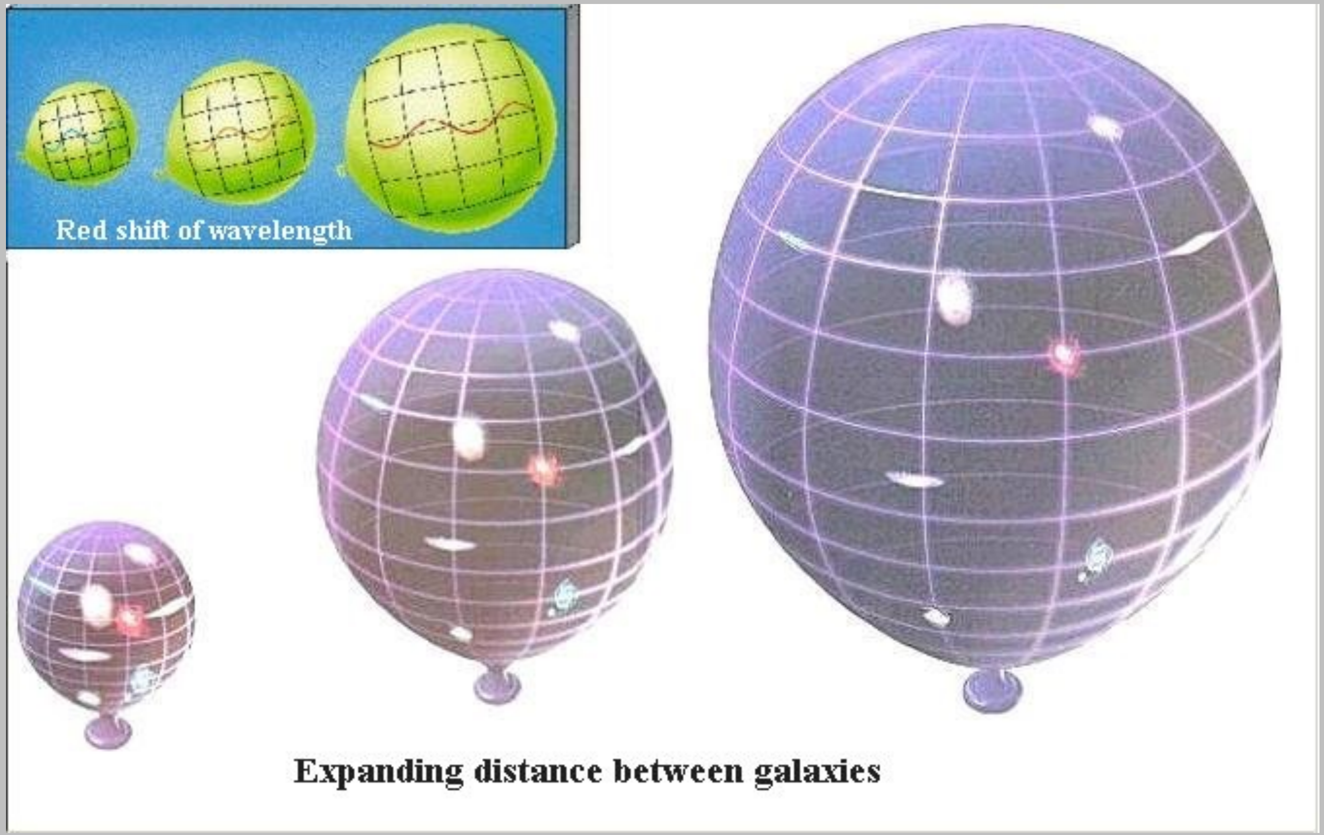
Coordinates and distances in an expanding universe

Do describe coordinates in an expanding universe, we define something called a **co-moving coordinate system** that expands along with the universe. We can then describe a **proper distance** this way

$$d = R(t)r$$

d : proper distance
r : co-moving distance
$R(t)$: dimensionless scale factor describing expansion

We define $t = t_0$ to be now, and $R(t_0) \equiv 1.0$. The scale factor of the universe is exactly one today by definition.



But the proper distance to a galaxy is not measurable in any conventional sense.

For these reasons, in cosmology we do not talk in terms of distances, we use redshifts.

It is not the distance you would have to travel to get to the galaxy.
It is also not the same as the distance the galaxy's light has travelled to get to us.
It is also not the distance you use in the magnitude-distance equation ($m - M = 5\log d - 5$)
It is also not the distance you use to convert angular size to physical size ($D_{phys} = d\tan\alpha \approx \alpha['']d/206265$)

Redshift and expansion factor

Start with the definition of redshift, writing in terms of the emitting wavelength of light and the observed wavelength of light.

$$z = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}}$$

$$z = \frac{\lambda_{obs}}{\lambda_{em}} - 1$$

Then rearrange terms a bit:

$$1 + z = \frac{\lambda_{obs}}{\lambda_{em}}$$

The stretching of light is due to the expansion of the universe, and is just given by the ratio of the scale factor. If the universe doubled in size, the wavelength was stretched by a factor of two.

$$1 + z = \frac{R_{obs}}{R_{em}}$$

And since we are observing today, and have defined the scale factor of the universe today to be exactly one:

$$1 + z = \frac{1}{R_{em}}$$

So scale factor is measurable!

If we detect light from a galaxy that has a redshift of $z = 3$, we are seeing the galaxy as it was when the scale factor was

$$\begin{aligned} R_{em} &= 1/(1+z) \\ &= 1/4 \end{aligned}$$

So we see the galaxy as it was when the universe was a quarter of its current size.

The age of the Universe

If the universe is expanding, then if we run it backwards it must have had a time when $R(t) \to 0$. The beginning!

When was that? Homer Simpson math time.

An object moving at a constant speed covers a distance

$$d = vt$$

or equivalently, to cover a distance d it will take a time

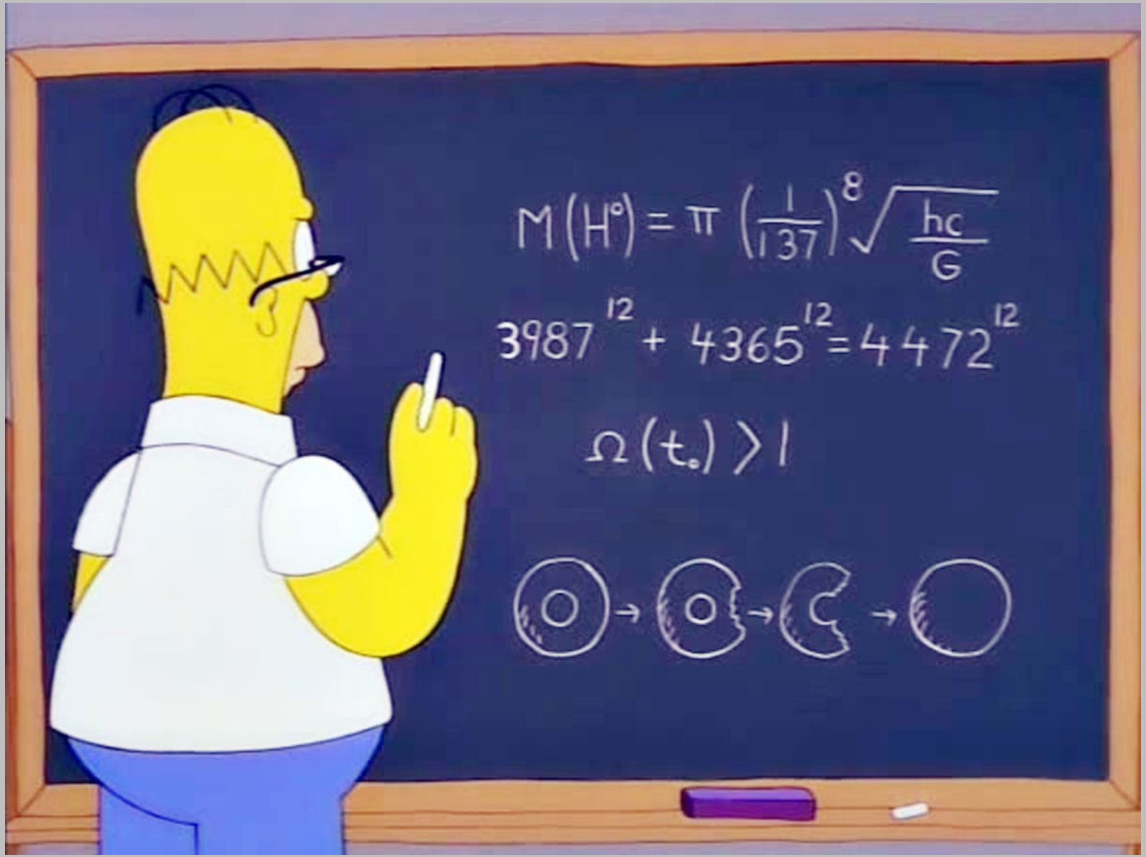
$$t = d/v$$

Now think of a galaxy at a distance d. If the expansion rate has been constant over time, that expansion rate is given by the Hubble constant: $v = H_0 d$.

Then $t_0 = d/v = d/(H_0 d) = 1/H_0$.

Homer says the age of the Universe is $1/H_0$.

What is the big assumption built in?
Constant expansion rate.

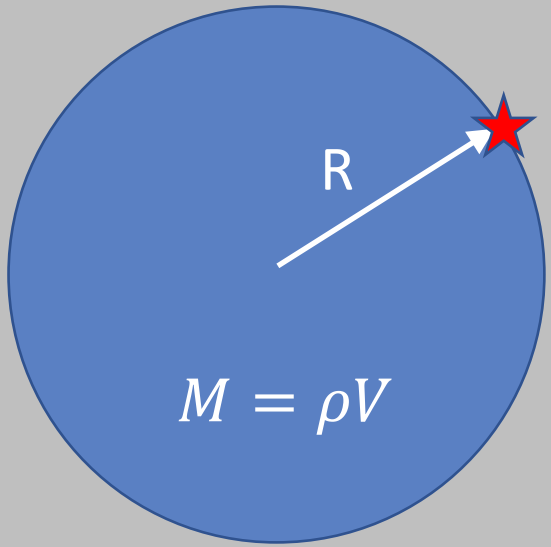


$H_0 = 72$ km/s/Mpc
$H_0 \approx 72$ pc/Myr/Mpc
$= 72 \times 10^{-6}$ Mpc/Myr/Mpc $= 7.2 \times 10^{-5}$ Myr^{-1}

Then $t_0 = 1/H_0 \approx 1.39 \times 10^4$ Myr = 13.9 billion years!

Solving for R(t): Newtonian Cosmology

Think of a particle on an expanding sphere with size R and total mass M. What is its equation of motion?



Notation:

R: distance from center (changes with time)

$\dot{R}$: 1st derivative w.r.t time (dR/dt, or velocity)

$\ddot{R}$: 2nd derivitive w.r.t time (d^2R/dt^2, or acceleration)

Also:

ρ : density of sphere (changes with time)

ρ_0 : density of sphere today (a fixed value)

$$\ddot{R} = -\frac{GM}{R^2} = -\frac{4\pi}{3} G\rho R$$

$\Leftarrow$ replacing M with $\rho \times V$

$$\rho = \rho_0 R^{-3}$$

$\Leftarrow$ density scales with volume

$$\ddot{R} = -\frac{4\pi}{3}\frac{G\rho_0}{R^2}$$

$\Leftarrow$ so substitute to get this

$$\dot{R}\ddot{R} + \frac{4\pi}{3}\frac{G\rho_0}{R^2}\dot{R} = 0$$

$\Leftarrow$ multiply both sides by $\dot{R}$ to get this

$$\frac{1}{2}\frac{d(\dot{R}^2)}{dt} + \frac{4\pi}{3}\frac{G\rho_0}{R^2}\frac{dR}{dt} = 0$$

$\Leftarrow$ substitute using this relation

$$\frac{d(\dot{R}^2)}{dt} = 2\dot{R}\ddot{R}$$

$$\frac{d}{dt}\left[\dot{R}^2 - \frac{(8\pi G\rho_0/3)}{R}\right] = 0$$

$\Leftarrow$ substitute using this relation

$$\frac{1}{R^2}\frac{dR}{dt} = -\frac{d(1/R)}{dt}$$

$$\dot{R}^2 - \frac{(8\pi G\rho_0/3)}{R} = -k$$

$\Leftarrow$ if $\frac{df}{dt} = 0$, then f is a constant

Solving for R(t): Newtonian Cosmology

so we had

$$\dot{R}^2 - \frac{(8\pi G \rho_0/3)}{R} = -k$$

rewrite this as

$$\dot{R}^2 = \frac{(8\pi G \rho_0/3)}{R} - k$$

to look at behavior over time.

Now, one final rewrite: replace ρ_0 with ρR^3 and divide everything by R^2 to get:

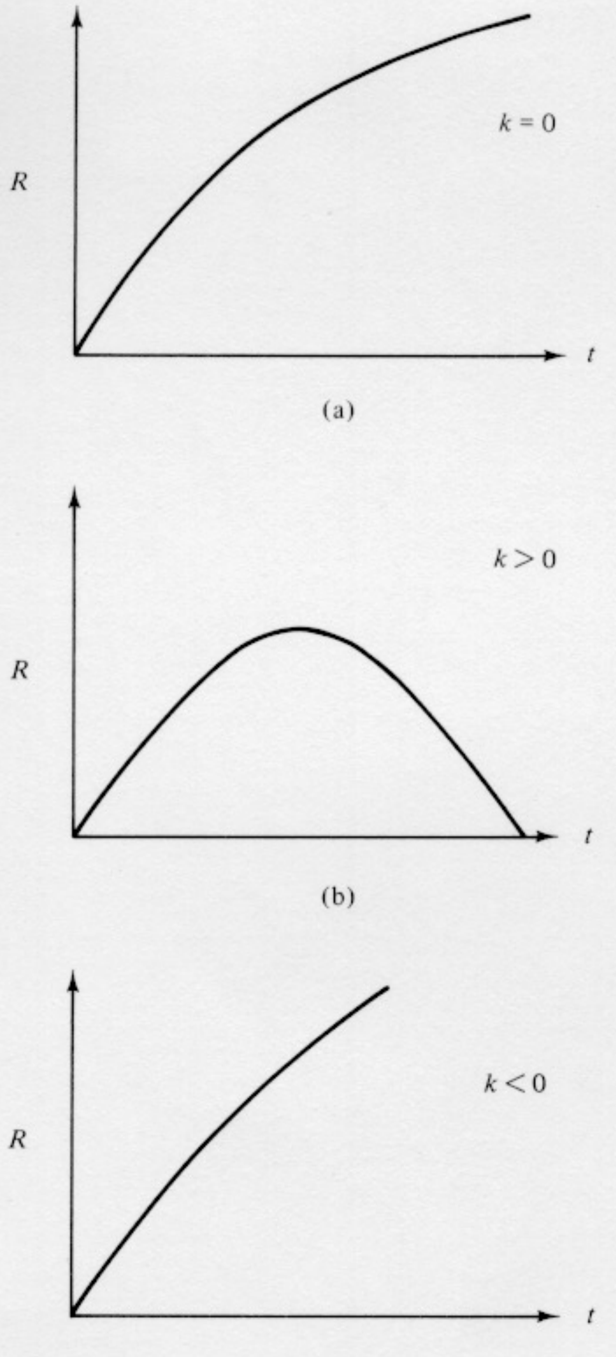
$$\left(\frac{\dot{R}}{R}\right)^2 - \frac{8}{3}\pi G \rho = -\frac{k}{R^2}$$

Possibilities for $\boldsymbol{k}$:

$\boldsymbol{k = 0}$: Then $\dot{R}$ is always positive, so the sphere always expands, but at an ever-slowing rate: $\dot{R} \rightarrow 0$ as $R \rightarrow \infty$.

$\boldsymbol{k > 0}$: $\dot{R}$ is initially positive but at some point will reach zero, then gravity wins and the sphere begins to contract.

$\boldsymbol{k < 0}$: $\dot{R}$ is always positive and so the universe is always expanding.



(a)
(b)
(c)

The Friedmann Equation

So under Newtonian dynamics we had

$$\left(\frac{\dot{R}}{R}\right)^2 - \frac{8}{3}\pi G\rho = -\frac{k}{R^2}$$

Surely a proper derivation using General Relativity can't look anything like that, right?

Actually, it can. Solving the Einstein field equations for an isotropic, homogenous universe gives the **dynamics equation**:

$$\frac{\ddot{R}}{R} = -\frac{4\pi}{3}G\rho + \frac{1}{3}\Lambda c^2$$

which solves to the **Friedmann Equation**

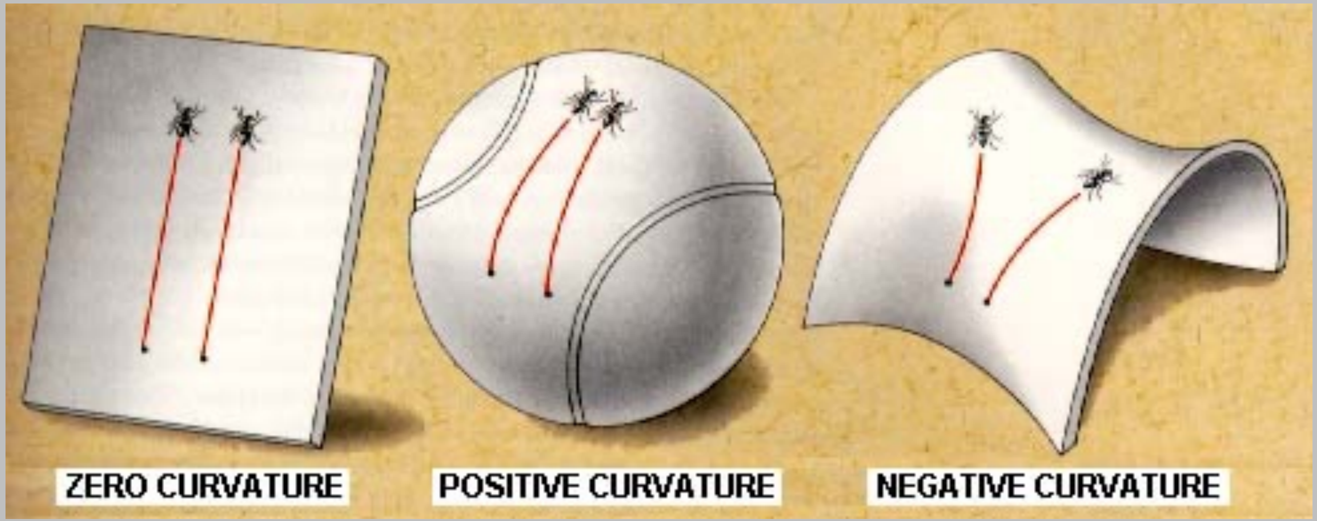
$$\boxed{\left(\frac{\dot{R}}{R}\right)^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2 = -\frac{kc^2}{R^2}}$$

Important Notes:

1. The dynamics equation says the without the cosmological constant, the universe can't be static. This is why Einstein introduced the cosmological constant: to get a static universe.

2. Under GR, k is linked to the **curvature of space**.

$k = 0$ $\qquad$ $k > 0$ $\qquad$ $k < 0$



3. *If there's no cosmological constant* ($\Lambda = 0$), the curvature of space and the expansion history are tightly connected.

$k = 0$	$k > 0$	$k < 0$
universe continually slows	universe eventually recollapses	universe expands forever

$$\boxed{\left(\frac{\dot{R}}{R}\right)^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2 = -\frac{kc^2}{R^2}}$$

The (Basic) Cosmological Parameters

The Hubble Parameter ($\boldsymbol{H}$):

$$H \equiv \frac{\dot{R}}{R}$$

H is the normalized rate of expansion, and changes with time as the universe expands.

Its value at the current time (i.e., at $t = t_0$), it is called the Hubble constant, H_0.

Current measures put $H_0 = 72$ km/s/Mpc or so.

$$\left(\frac{\dot{R}}{R}\right)^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2 = -\frac{kc^2}{R^2}$$

The (Basic) Cosmological Parameters

The matter density parameter ($\boldsymbol{\Omega_m}$):

Rewrite the Friedmann equation using the Hubble parameter and setting $\Lambda = 0$:

$$H^2 - \frac{8}{3}\pi G\rho = -\frac{kc^2}{R^2}$$

$\mathrm{k} = 0$ means the universe is spatially flat, so a "no lambda" universe ($\Lambda = 0$) universe is spatially flat ***if*** it has a critical density ρ_{crit} given by:

$$\rho_{crit} = \frac{3H^2}{8\pi G}$$

We define the matter density parameter as

$$\Omega_m = \frac{\rho}{\rho_{crit}}$$

Current measures put $\Omega_{m,0} \approx 0.30$

The matter density parameter tells you if gravity alone is sufficient to "flatten" the Universe (i.e., if $\Omega_m = 1$).

The data says it is *not*.

$$\boxed{\left(\frac{\dot{R}}{R}\right)^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2 = -\frac{kc^2}{R^2}}$$

The (Basic) Cosmological Parameters

The “dark energy” density parameter (Ω_Λ):

Rewrite the Friedmann equation using the Hubble parameter and setting $\rho = 0$:

$$H^2 - \frac{1}{3}\Lambda c^2 = -\frac{kc^2}{R^2}$$

$\mathrm{k} = 0$ means the universe is spatially flat, so a matter-free universe ($\rho = 0$) universe is spatially flat if it a critical value of Λ of

$$\Lambda_{crit} = \frac{3H^2}{c^2}$$

We define the dark energy density parameter as

$$\Omega_\Lambda = \frac{\Lambda}{\Lambda_{crit}}$$

Current measures put $\Omega_{\Lambda,0} \approx 0.7$

The dark energy density parameter tells you if Λ alone is sufficient to “flatten” the Universe (i.e., if $\Omega_\Lambda = 1$).

The data says it is *not*.

$$\left(\frac{\dot{R}}{R}\right)^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2 = -\frac{kc^2}{R^2}$$

The (Basic) Cosmological Parameters

"Total Omega" (Ω): $\boldsymbol{\Omega = \Omega_m + \Omega_\Lambda}$

Total Omega tells you if gravity and dark energy combined can make the universe spatially flat (if $\Omega = 1$):

Current estimates say $\Omega_0 = \Omega_{\mathrm{m},0} + \Omega_{\Lambda,0} \approx 0.3 + 0.7 = 1.0$

The universe is spatially flat!