ASTR 222 Homework #4

Working with the Tully-Fisher Relationship (20 points)

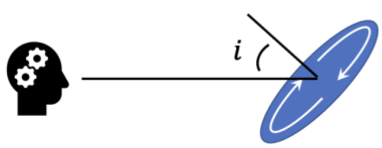
The Tully Fisher Relationship for spiral galaxies can be expressed by $M = a\log\left(\frac{V_c}{300\ km/s}\right) + b$, where M is the absolute magnitude of the galaxy (in some filter band), V_c is the galaxy's rotational velocity, and a and b are the calibrated parameters of the relationship. Expressed this way, b is actually the absolute magnitude of a galaxy that rotates at 300 km/s. Note also that since absolute magnitudes depend on which band you observe in (say, B or V), the fitted parameters a and b will also differ depending on which band you are working in.

In general, unless we know the distances of galaxies, we can't make a Tully-Fisher plot, since we don't know absolute magnitudes without knowing distances. But we can be crafty and realize that if we look at galaxies in within a single galaxy cluster we can plot **apparent** magnitude against $\log\left(\frac{V_c}{300\ km/s}\right)$ and still get the proper slope (a). Explain why this is.

So now here is a Tully Fisher dataset for galaxies in the Virgo Cluster (from Pierce & Tully 88). The dataset has the following information:

- the NGC catalog number of each galaxy,
- the **apparent magnitude** of the galaxies in blue (B), red (R), and near-infrared (I) filters.
- the **inclination** of the galaxy to our line of sight (in degrees, where $inc = 0°$ means the galaxy is seen face-on and $inc = 90°$ means the galaxy is seen edge-on).
- the **observed rotation speed** ($V_{c,obs}$) of the galaxies in km/s.

First, we need to make an inclination correction to convert the observed rotation speed ($V_{c,obs}$) to the true rotation speed (V_c) of the galaxy. Use the sketch below to work out an trigonometric function for the correction (show your work), then apply this correction to the observed rotation speeds to get the true rotation speed.



Now make a Tully-Fisher plot (y-axis: apparent magnitude m, x-axis: $\log\left(\frac{V_c}{300\ km/s}\right)$) for each of the B, R and I bands, and each case, fit a line to the observed Tully-Fisher relationship: $m = a\log\left(\frac{V_c}{300\ km/s}\right) + b'$, where b' is different from b, since we are using apparent magnitudes instead of absolute magnitudes. For the fit in each band, give the slope (a) and its uncertainty, the intercept (b') and its uncertainty, and the scatter in the fit.

Give physical arguments about which band would best define the Tully-Fisher relationship. Think both about stellar populations and dust.

Now we need to calibrate the Tully-Fisher relationship. We want to know how absolute magnitude depends on circular velocity, which means we need to know a distance to the Virgo cluster. Using the Hubble Space Telescope, we can use Cepheid variables in the spiral galaxy M100, also known as NGC 4321. Here are the reduced light curves (from Freedman+ 94). These are plots (one for each detected Cepheid) of apparent magnitude on the y-axis and "phase" (ϕ) on the x-axis. ("Phase" means time relative to the period of the variable star, so $\phi = 0.5$ means halfway through the period).

From the plots, estimate the average apparent magnitude of each Cepheid (an eyeball estimate is fine!), then use the Cepheid period-luminosity relationship ($M = -2.43 \log P - 1.62$, where P is the period in days) to calculate a distance estimate to M100 for each Cepheid. Average all those estimates together to give your best estimate for the M100 distance. Also give a statistical uncertainty to your distance.

Now use this distance to calibrate the Tully-Fisher relationship. You had a T-F relationship that connected apparent magnitude (m) with $\log V_{c,true}$, and now you know the distance to Virgo to turn apparent magnitude (m) into absolute magnitude (M), so you can rewrite your T-F relationship now in terms of absolute magnitude: $= a \log\left(\frac{V_c}{300\ km/s}\right) + b$. What are a and b, and their uncertainties (σ_a and σ_b)? Remember now that the uncertainty in b should incorporate your uncertainty in both the fit and in the Virgo distance estimate.

What do you feel are the main sources of uncertainty, both random (i.e., the error in fitting and calculations) and systematic (the uncertainty due to the over conceptual method), in your derivation of the full calibrated Tully-Fisher relationship?

Now imagine you are looking at a spiral galaxy in the Coma cluster. It has an I-band **apparent** magnitude of $m_I = 13.5$, an **observed** rotation speed of $V_c = 180$ km/s, and an inclination of $i = 65°$. What is the galaxy's absolute magnitude, and its uncertainties (propagate the uncertainties in the T-F fit (σ_a and σ_b) to get the uncertainty on the absolute magnitude). From this absolute magnitude estimate, work out the distance to Coma and the uncertainty in that distance. To get that uncertainty in distance, use the approximation that the relative uncertainty in distance (σ_d/d) is roughly half the magnitude uncertainty in the distance modulus.

Donkey Dark Matter (10 points)

One of my favorite ideas about the infamous "dark matter" surrounding galaxies is that it is made up of a population of free floating space donkeys (FFSDs). FFSDs would not radiate in the optical (I mean, come on, when you look at a donkey, it's not glowing, right?) but would emit light in the infrared (since they would have to have little heat generators in the donkey space suits to keep them warm). So let's see if we can rule out this model.

Say the dark matter halo of a bright spiral galaxy like the Milky Way has a mass of $\mathcal{M} = 10^{12}\ \mathcal{M}_\odot$. If it was made of FFSDs, what would the bolometric (total) luminosity of the dark matter halo be? (Assume FFSD are blackbody radiators.) What would the peak wavelength of this light be? Compare this to the bolometric luminosity of the Milky Way stars ($\approx 5 \times 10^{10}\ L_\odot$). How many times brighter or fainter are the FFSDs than the Milky Way's stars? Do you think we'd be able to detect them?

The Mass of the Coma Cluster (20 points)

In this problem we are going to estimate the total mass of the Coma cluster of galaxies. For this problem, adopt a Hubble constant of $H_0 = 72$ km/s/Mpc.

First, go look at an image of the Coma cluster online, so that you know what you are studying.

Now here is a dataset of galaxies in a 6x6 degree field around the Coma cluster (from Doi+ 95). It contains

- name: galaxy ID number
- x & y: angular position relative to the cluster center of Coma (defined by NGC 4886), measured in arcminutes
- cz: the observed radial velocity of the galaxy in km/s. cz=0 means no measurement.
- Bt: the blue (B) magnitude of each galaxy

First make an x,y plot of the galaxy distribution. (make sure the axes on your plot have a square aspect ratio; use `set_aspect('equal')` when making your plot!) See -- it's a cluster!

Now make a histogram of the radial velocity of all the galaxies (use bins with width of 250 km/s, and don't include galaxies without velocity measurements when making your histogram!). Explain how this plot would help you decide which galaxies actually were part of the Coma cluster?

Looking only at galaxies with $4{,}000 < cz < 10{,}000$ km/s (explain why this cut is important), calculate the mean velocity and velocity dispersion (σ_v) of the galaxy sample. From your data, how far away is Coma (in Mpc)?

Now calculate the total blue luminosity (in solar luminosities) for all the Coma cluster galaxies combined. For this part, ignore galaxies with $cz < 4{,}000$ km/s or $cz > 10{,}000$ km/s, but include galaxies with unmeasured redshifts (since they are probably in the cluster). Remember, *don't add magnitudes!* Calculate the luminosity of each galaxy individually, then add the luminosities of the galaxies. For reference, the absolute blue magnitude of the Sun is $M_{B,\odot} = +5.5$.

Next figure out the radius (in arcminutes) which contains roughly half the total blue luminosity of the cluster (it doesn't have to be an exact solution, but you should get it to an accuracy of +/- 20 arcminutes or so). This is called the half-light radius, which is our estimate of the size of the Coma cluster. What is the half-light radius of Coma in Mpc?

Finally calculate the total mass of Coma (in solar masses), using the following expression for cluster mass: $\mathcal{M}_{tot} = 5\langle R\rangle\sigma_v^2/G$. Use your half-light radius (in parsecs!) as your estimate of $\langle R\rangle$, the characteristic size of the cluster.

If the stars in the Coma cluster galaxies have a stellar mass-to-light ratio of $(\mathcal{M}/L)_* = 3\ \mathcal{M}_\odot/L_{B,\odot}$, what is the total mass of stars in the Coma cluster?

If the galaxies in the Coma cluster have a total mass-to-light ratio of $(\mathcal{M}/L)_{tot} = 20\ \mathcal{M}_\odot/L_{B,\odot}$, what is the total galaxy mass of Coma (this will consist of stellar mass and the mass of any dark matter and gas which is contained inside galaxies).

X-ray measurements indicate that the Coma Cluster has a hot gas mass of $\approx 3 \times 10^{14}\ \mathcal{M}_\odot$. What fraction of Coma is dark matter unassociated with galaxies (i.e., is dark matter distributed smoothly throughout the cluster)?

ChatGPT (10 points)

ChatGPT is online at https://openai.com/blog/chatgpt

Give ChatGPT an ASTR221/222 exam-level question about stars or galaxies and evaluate its response. How accurate is it? How much physical/technical detail did it go into? Give its answer a letter grade.

Then also dig a bit deeper and get a bit more technical. Ask some follow-up questions about the subject, and ask it for citations to scientific journal articles. Ask it for a link to that citation. If it gives you a link, see if the linked paper seems like a plausible source (does it discuss the topic, for example?).

Then throw it a little bit of a curve ball: ask it something about new data or a new discovery of something that you have completely made up. Don't make up a question that's obviously crazy (like unicorns on Pluto), but is "sounds" reasonable but is not actually real. See how it responds. If it talks about the discovery (or a related issue), ask it for citations and links, and click on those. See how they relate to what you asked about.

In short, play around a bit and see what ChatGPT does well, and where/how it behaves poorly. Write a couple of paragraphs describing what you did and giving your assessment of its performance.

See this transcript of mine to get a feel for what you could try. Here are some possible prompts you could start with, or better yet you could make up your own:

- Describe Cepheid variable stars and how they can be used to get distances to galaxies.
- Explain why galaxies have spiral arms.
- What happens when two elliptical galaxies merge?
- Describe the Oort limit for disk galaxies.
- Describe the discovery of an active nucleus in the galaxy M101.

There are many many other possibilities you could try! Have fun with this one….

Note: using ChatGPT does involving creating an account on their system. It's pretty simple to do without giving much personal information, but if you do not want to do that, let me know and I'll do the querying, etc for you and simply send you the transcript to evaluate. But its more fun to do it yourself and interact with the AI on the fly…..